An Agent-Based Model of the Automobile Insurance Market with an Endogenous Underwriting Cycle

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Abstract

We develop an agent-based model of the automobile insurance market, characterized by heterogenous and boundedly rational insurers in a monopolistically competitive market differentiated by non-price characteristics over which buyers have preferences. This contrasts with conventional economic models with homogeneous, representative insurers, sharing a common goal of profit maximization. Our model is calibrated and validated on UK insurance data. Time series analysis demonstrates that cycles in market loss ratios emerge endogenously, even in the absence of factors such as capital shocks and interest rate fluctuations.

Keywords: Insurance cycle, Monopolistic competition, Bounded rationality, Agent-based model

1. Introduction

Insurance cycles are a phenomenon of property-liability insurance markets, whereby levels of profitability, prices and coverage are observed to rise and fall periodically (Harrington, Niehaus and Yu, 2014; Weiss, 2007). In this paper, we propose an explanation for the propagation of these cycles and, for this purpose, we concentrate on the automobile insurance market. Our model has four distinguishing characteristics: the absence

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of exogenous factors; the absence of institutional and regulatory interventions; imperfect competition; and bounded rationality. The model is implemented and simulated as an agent-based model (ABM).

First, we assume that there is *no exogenous factor* driving insurance cycles, except for random uncorrelated claims on automobile insurance policies. The most established hypothesis explaining the insurance cycle is the capital constraint hypothesis (Winter, 1988, 1994; Gron, 1994; Cummins and Danzon, 1991; Niehaus and Terry, 1993). This suggests that major industry-wide loss shocks or interest rate shocks (Doherty and Kang, 1988; Doherty and Garven, 1995), in combination with short-term constraints on the supply of capital to the insurance industry, are the catalysts for cycles. Such shocks are precluded in our model.

Second, we assume the *absence of institutional and regulatory interventions*, including solvency constraints. Irrational forecasting methods (Brockett and Witt, 1982; Venezian, 1985) and information delays (Cummins and Outreville, 1987) have been proposed as a possible cause for cycles, but these are ruled out in our model. In some models, solvency constraints may be explicit and imposed by regulation. In other models, such as the financial quality hypothesis model, where it is assumed that insurance demand falls with worsening insolvency risk, these constraints may be implicit (Harrington and Danzon, 1994; Cagle and Harrington, 1995). In this paper, we assume that insurers have sufficient capital and remain solvent.

It may come as a surprise that insurance cycles can arise without such shocks and constraints. Our finding is that the third and fourth characteristics of our model—that the insurance market is imperfectly competitive and that insurers exhibit boundedly rational pricing behavior—are sufficient to generate cyclical dynamics.

Imperfect competition stems from information asymmetries, heterogeneity among insurers and their products, and insureds' preferences on product attributes other than price. Whilst property-liability insurance markets, and in particular the automobile insurance market, are very competitive, we argue that the perfect competition assumption is an economic idealization. In this sense, our work is close in spirit to that of Harrington and

Danzon (1994) and Harrington et al (2008), whose model also permits heterogeneous insurers and price dispersion. Our monopolistic competition model is inspired by Stiglitz (2004), Salop (1979), and Schlesinger and Schulenburg (1991). In our model, customers respond not just to price but also to the subjective, nonmeasurable perceptions that they have regarding the non-price characteristics of insurers and their products. For example, insurance buyers will have views on an insurer's trustworthiness, size, reliability of service, financial solvency etc., based on their personal experience and their exposure to marketing. The vast amounts spent on marketing and advertising in the insurance sector suggest that these perceptions matter.

The fourth key feature of our model is that insurers' actual pricing behavior is *boundedly rational*. Since the work of Cummins and Outreville (1987), insurers have been assumed to have rational expectations. Casual inspection of insurers' pricing departments suggests, however, that insurers practice cost-based pricing, which violates the rational expectations norm and results in mispricing. Unlike Brockett and Witt (1982) and Venezian (1985), we suggest that it is pricing activity, rather than actuarial costing, which is subject to bounded rationality. Imperfect information about their customers' non-price preferences means that insurers cannot perfectly read their customers' preferences and can only estimate approximately the elasticity of demand for their product.

The combination of an imperfectly competitive market and bounded rationality is enough to sustain cycles in pricing across the insurance industry. These cycles arise endogenously in our model. *First*, random shocks to claim costs result in shocks to the cost-based prices charged by an insurer. If uncorrelated adverse shocks occur to enough insurers, at random, these insurers will raise their prices, causing the average price per unit of insurance risk to increase. Initially, insureds tend not to switch to other insurers because the difference in prices among insurers is small compared to the additional 'cost' of choosing an insurer whose non-price characteristics they dislike. Boundedly rational pricing means that insurers underestimate price-elasticity of demand, and consequently increase their mark-ups on cost. Prices can therefore keep rising. *Second*, at high enough prices, buyers begin to respond more to price than before, and less to non-price attributes than before. They are more likely to switch to other insurers. Random differential effects cause customers to migrate to comparatively cheaper insurers, overcoming the greater affinity that customers have for their current insurer. The market becomes more competitive on price than before, and this eventually forces prices down. *Third*, insurers cut prices further than needed, even as prices are falling, because they underestimate the price-elasticity of demand again, but this time in the lower portion of their demand curve. This accelerates the downswing in prices. *Fourth*, when prices reach a low enough level, the market becomes more competitive on non-price factors, and less competitive on prices, than before. Insurers remain misinformed about how non-price attributes govern their customers' preferences, which causes them to misjudge their pricing. They believe that lower prices will attract customers, so they end up pricing below fair premium levels for a prolonged period, until the cycle turns and starts anew.

We stress that our model contradicts neither the capital constraint hypothesis nor the financial quality hypothesis, but complements them. In the first (rising) part of the cycle described above, correlated industry-wide shocks could also trigger the start of an insurance hard market. Such shocks will affect all insurers. Again, customers do not switch because it is the relative difference among the insurers which matters. In the third (falling) phase of the cycle above, a concern for financial quality could shift the demand curve. We model neither of these effects here. Furthermore, we disregard availability of coverage—acute shortages of coverage occur during 'insurance crises'—since we are concerned with automobile insurance which features some form of regulatory compulsion.

In the next section of this paper, we review the literature that is germane to our model. In particular, we review current theories of the insurance cycle, of bounded rationality and of imperfect competition. We provide a formal model specification in the section which follows. Our modeling estimation and validation procedures are explained in the subsequent section. The results from our model are described, discussed and we close with some final remarks.

2. Motivation and Related Literature

In this section, we provide a motivation for the assumptions in our model and we describe how they either reflect or depart from existing work.

2.1. Insurance Cycles

The absence of exogenous factors is an important feature of our model. We do not allow for catastrophic insurance loss shocks. Such shocks are a feature of the capital constraint hypothesis (Winter, 1988, 1994; Gron, 1994; Cummins and Danzon, 1991; Niehaus and Terry, 1993) which assumes that markets are imperfect and that the short-run supply of capital to insurers is limited, particularly just after catastrophic industry-wide events when uncertainty about losses in the insurance industry is at its highest. Such events strain insurers' reserves, causing them to seek to shift the cost of capital shocks to policyholders by raising premiums and cutting coverage.

An alternative explanation for insurance cycles is that they are caused by variations in interest rates. Since the insurance premium is based on the expected discounted value of future claims and expenses, prices will vary inversely with interest rates. Doherty and Kang (1988) and Fields and Venezian (1989) show that a cycle in interest rates might explain the cycle in profitability in the insurance industry. Doherty and Garven (1995) reason that interest rate changes will significantly affect the surplus of an insurer whose assets and liabilities are not duration-matched, thereby generating a capital shock. In the presence of capital constraints, a negative shock will lead to prices increasing, while a positive shock explains falling prices.

In our model, we preclude all capital shocks, whether from catastrophic losses or interest rate variability, and we assume a constant risk-free rate of interest. Our motivation for excluding exogenous shocks is to achieve parsimony and to avoid mining a class of models for one which injects more or less regular shocks, thereby generating a disturbance away from equilibrium with resultant dynamic behavior. This is not to say that capital shocks do not contribute to insurance cycles and crises. Rather, we think that a first-stage analysis of the insurance market under imperfect competition and bounded rationality should exclude these outside forces. Indeed, we find that cycles in insurance markets can arise without such shocks because of the structure of an imperfectly competitive market and the bounded rationality of insurers.

We have a similar motivation for ignoring institutional interventions and solvency constraints in our model. Institutional factors which cause informational delays and adjustment costs, such as accounting and reporting lags, are shown to be a possible cause for underwriting cycles by Cummins and Outreville (1987), as well as by Lamm-Tennant and Weiss (1997) in an international context. We also ignore the effect of rating agencies on insurer behavior. Doherty and Phillips (2002) show that insurers have increased their reserves in response to more stringent rating standards in the past. As for solvency, it is an important factor in the capital constraint hypothesis. In the original version of this hypothesis (Winter, 1988; Gron, 1994), the risk of breaching regulatory solvency limits impels insurers to raise premiums and underwriting standards in order to raise capital. Solvency is also an important component of the risky debt hypothesis of Cummins and Danzon (1997). A firm with greater capital, and hence lower insolvency risk, is more attractive to insurance consumers and it can price policies more highly than a comparable firm with lower capital. Capital shocks with solvency constraints again drive insurance cycles, but in the opposite direction compared to the capital constraints model.

We emphasize again that we ignore solvency and institutional interventions in our model because we wish to exclude external effects, as a first step, and focus on the core dynamics of the insurance market. This enables us to bypass the unsettled question of whether insurance prices are directly or inversely related to surplus (Choi, Hardigree and Thistle, 2002; Weiss and Chung, 2004). This also means that our model is predicated on neither the capacity constraint hypothesis nor the risky debt hypothesis. In fact, we do not assume the a priori existence of underwriting cycles at all: tests to identify these cycles have recently been criticized by Boyer, Jacquier and Van Norden (2012) and Boyer and Owadally (2015).

2.2. Imperfect Competition, Non-Price Preferences and Price Dispersion

A central thesis of our paper is that property-liability insurance markets are very competitive, but not perfectly competitive. It seems clear that the automobile insurance market is not oligopolistic, since it does not consist of only a few firms. In this context, the game-theoretic approach of Dutang, Albrecher and Loisel (2013) and Hardelin and Lemoyne de Forges (2012) does not yield cycles. We posit instead that monopolistic competition (Stiglitz, 2004) is a suitable model for competition on the automobile insurance market.

The monopolistic competition assumption is not new. It underlies the excessive price-cutting model of Harrington and Danzon (1994) and Harrington et al (2008). Their model addresses a weakness of the capital constraint hypothesis, which explains hard markets (when profitability and prices rise) and insurance crises (when coverage falls drastically), but is less satisfactory when it comes to the causes of soft markets (when prices are persistently depressed below fair premium levels). Harrington and Danzon (1994) include insurer heterogeneity in terms of intangible capital (e.g. franchise value) and the quality of loss forecasts, and find that weaker insurers have little to lose by undercutting more established insurers. This forces the latter also to cut prices to compete, thereby triggering a down-swing in the underwriting cycle. This shows that insurer differentiation, imperfect competition and interaction between insurers are contributory factors to underwriting cycles, consistent with the view of insurance analysts such as Feldblum (2001).

A monopolistically competitive market must satisfy a number of properties (Samuelson and Nordhaus, 1998). First, there are clearly a large number of insurers selling this type of insurance. Second, insurance firms have some degree of control on their pricing in the short run. Third, informational asymmetries, in the form of adverse selection and moral hazard, are clearly present. Fourth, there are few entry and exit barriers for firms on the market.

Monopolistic competition also requires product differentiation, either real or perceived (Samuelson and Nordhaus, 1998, p. 158). Schlesinger and Schulenburg (1991) emphasize that, although insurance contracts may be fairly similar from one insurer to another, the insurance product is a service and is heterogeneous. There are pronounced differences in consumer perceptions of qualities such as insurer's reliability and trustworthiness (Wells and Stafford, 1995). There are also numerous actual differences among insurers: reputation and branding, marketing and advertising methods, distribution channels, website design, sales promotional activity, payment methods, ease of policy cancellation, out-of-hours telephone service, claims service, communication systems and access to information, data protection and safety etc. Doerpinghaus (1991) also analyzes complaint handling which can vary from insurer to insurer in the automobile insurance market. Crosby and Stephens (1987) investigate the insurer-customer relationship (albeit in the context of the life insurance industry). Such widespread product differentiation clearly contributes to the price dispersion that is observed on the auto insurance market.

There are several models of monopolistic competition, other than the one underlying Harrington and Danzon (1994) and Harrington et al (2008). The Dixit-Stiglitz model is the best known (Dixit and Stiglitz, 1977), but it captures individuals' desire for product diversity and is inapplicable in the context of automobile insurance. The model of Salop and Stiglitz (1977) imposes costs due to information asymmetry, and does generate price dispersion as well as cycles. In this paper, we use an implementation, within an agent-based model, of the economic location model of Salop (1979), also used by Schlesinger and Schulenburg (1991) in insurance. Distance between locations measures the affinity between a customer's non-price preferences and the insurer's characteristics. In related models in the insurance literature, distance is used to measure search costs and thus used to explain price dispersion (Berger, Kleindorfer and Kunreuther, 1989). Note that this is related to, but distinct from, the switching costs assumed Doherty and Posey (1997) and others. It may also be possible to calibrate the distance on the location space using a customer's product experience, and to include quality as a vertical differentiation variable, as in Riordan (1986), but we do not do this here.

2.3. Bounded Rationality and Cost-Based Pricing

Another key assumption in our model is that insurers are boundedly rational in their pricing behavior. This is a departure from the rational expectations norm which is used in virtually all models of underwriting cycles since the influential work of Cummins and Outreville (1987). Bounded rationality refers to the limited cognition, limited information and limited time which agents have to address complex decision-making problems. Costless optimization with perfect information is not available in practice, so these limited resources motivate agents to use simpler cognitive strategies. Bounded rationality models come in two main flavors (Harstad and Selten, 2013; Mallard, 2012; Pingle, 2004; Conlisk, 1996). One class of models involves sub-optimization, e.g. optimization with constraints on information or on decision variables, and optimization with deliberation or information processing costs. The other class is based on evidence-based characterizations of decision-making, and includes satisficing, quantal response equilibrium models in game theory, and the use of heuristics. In our model, we use the heuristic approach to describe realistically the practice of pricing by insurers.

Heuristics are shortcuts, or rules of thumb, which simplify the computation of a decision problem (Mallard, 2012; Conlisk, 1996). In their authoritative survey of behavioral corporate finance, Baker and Wurgler (2013) state that "Boundedly rational managers cope with complexity by using rules of thumb that ensure an acceptable level of performance and, hopefully, avoid severe bias" and "Bounded rationality offers a reasonably compelling motivation for the financial rules of thumb that managers commonly use." Heuristics can lead to behavioral biases or judgment errors, but many economic psychologists contend that simple "fast-and-frugal" heuristics can also be efficient and accurate problem-solving tools, given the limits of human cognition (Todd and Gigerenzer, 2003). This may be one reason for their widespread use by corporate managers and individual investors. Although the models of bounded rationality differ from those of behavioral economics, the compendium of evidence presented by Kunreuther, Pauly and McMorrow (2013) does support the absence of full rational behavior by insurers, and thus supports our model assumption.

In our model, bounded rationality is implemented through cost-based or mark-up pricing (Conlisk, 1996; Baumol and Quandt, 1964). Insurers use a mark-up heuristic to maximize profits approximately by adjusting actuarial cost by a factor which depends on their estimate of the price-elasticity of demand for their product. Management accounting studies report that cost-based pricing is very prevalent among firms in a wide range of industries. For example, Drury and Tayles (2006) report that 60% of 112 firms surveyed in the U.K. use it in some form to set prices. Many firms practise an approximate form of cost-based pricing, with a rule of thumb rather than a precise calculation of marginal cost and elasticity of demand. There is well-established evidence that cost-based pricing happens in practice in insurance: an 'actuarial cost' is calculated based on past claims data and actuarial premium principles (Kaas et al., 2008); this is regarded as an indication and is subsequently adjusted by underwriters in the light of market competition (Harrington and Niehaus, 2003, p. 137). The statistical evidence presented by Choi, Hardigree and Thistle (2002) lends support to an actuarial cost underpinning insurance prices. Insurance analysts such as Booth et al. (2005, p. 404) are explicit that insurance pricing consists of a two-stage process, the first being a costing exercise with actuarial input and the second being a pricing stage which involves a commercial adjustment to the cost. Our modeling assumption of bounded rationality through cost-based pricing is therefore well-founded in practice.

There are three further points worth stressing regarding cost-based or mark-up pricing. First, the insurers in our model attempt to maximize expected profit, and are risk-neutral in this sense. The insurers utilize a realistic mark-up pricing heuristic, rather than solve a stochastic dynamic optimization problem, which never happens in practical settings since market conditions are rarely as idealized and simplified as would be required in such optimization problems. Indeed, one reason for the widespread use of simple heuristics may be that collective experience in organizations teaches managers that some heuristics can deliver robust and near-optimal performance. For example, Conlisk (2003) shows that adaptive heuristics for stopping searches, based on an aspiration target which involves an average of past performance, can approximate optimal stopping rules. Owadally et al. (2012) also show that a simple rule based on deviations from a target can lead to robust contribution rules for a savings and investment plan. Baumol and Quandt (1964) investigate rules of thumb which managers may use to achieve "optimally imperfect" pricing decisions in practical settings and would refer to cost-based pricing as a "pseudo-maximizing" rule for pricing.

Second, in our model, it is *managers' pricing activity*, rather than the *actuarial costing* exercise, which suffers from bounded rationality. This is therefore distinct from the early work by Brockett and Witt (1982) and Venezian (1985) which shows that quasi-Bayesian *actuarial* forecasting of future insurance claim costs (which flouts rational expectations) can result in a second-order autoregressive process in loss ratios.

Finally, cost-plus pricing may be criticized as simplistic, but it is also regarded as a method which enables managers to determine optimal prices efficiently in a practical setting. See for example Petersen and Lewis (1999, p. 429) and Hirschey and Pappas (1996, p. 640). It is expensive and difficult to collect information on demand schedules, marginal costs and marginal revenues, so cost-plus pricing represents a pragmatic proxy for marginal analysis. Insurers have limited information about demand schedules and have uncertain knowledge of consumers. Indeed, Warthen and Somner (1996) refer to the elasticity of demand for individual insurers as a variable that cannot be estimated. Monopolistic competition means that firms are interdependent and will adapt their strategies in response to the market environment and to each other, so the demand function faced by any one firm is highly variable and not easy to measure. A form of cost-plus pricing is therefore more practical than a comprehensive marginal analysis of revenue and cost.

2.4. Agent-Based Modeling

It is difficult to model imperfect competition and bounded rationality in the standard rational-choice economic framework. We therefore use an *agent-based model* (ABM) as the tool for our analysis (Tesfatsion and Judd, 2006). This involves micro-simulation of the behavior of each insurer, building a simplified insurance market from the bottom up. ABMs have been successful at replicating stylized facts of stock markets, such as heavy-tailed returns and long memory, as well as in modeling contagion in financial markets, pricing on energy markets and other financial phenomena (Noe et al., 2003; Banal-Estañol and Micola, 2009; Aymanns and Farmer, 2015; LeBaron, 2000).

All agent-based models have a number of common features. First, agents (in our case, insurers) follow simple rules of behavior, but these rules are allowed to differ, thereby giving rise to heterogeneity. This contrasts with neo-classical economic models with a single representative agent or with infinite homogeneous and rational agents. Second, all markets exhibit structure and heterogeneous agents interact locally within this structure. In general, this may happen because of information barriers, geographic effects, time lags, product differences etc., but in our case we argue that the non-price attributes of insurers, and customers' preferences over these attributes, creates heterogeneity and a diffuse segmentation of the automobile insurance market. Third, agents' behavior and interaction may result in feedback which is positive and self-reinforcing, or negative and self-balancing. Although agents follow simple behavioral rules, interaction and feedback creates a system which behaves qualitatively differently from the way in which the individual agents themselves behave. This bottom-up non-reductionist approach allows for the emergence of complex behavior—such as cycles or extreme events—which cannot be predicted by studying the individual elements of the model. Finally, agents adapt and learn about their competitive environment over time. In our model, insurers are continually learning about their customers' price-elasticity of demand.

3. Model Specification

Basic assumptions. Our agent-based model of the automobile insurance market involves a simplified insurance market, with insurers and customers only, excluding reinsurers, brokers and intermediaries. Time is discrete, the unit of time being a policy year. There are fixed numbers of insurers (N) and customers (M). It is compulsory for all customers to purchase an insurance policy and they cannot cancel their policy mid-year. Customers are myopic in that they care about insurance purchase costs over the next period only, disregarding future periods. Claims occur independently and are identically distributed, across all customers. There is zero claim inflation. Tax and administration expenses are disregarded. There is no moral hazard or fraud.

Outline of market process. At the start of every year, each insurer offers its own unique market-competitive price to all customers. Every customer calculates the 'total cost' of purchasing insurance from every insurer. This includes both the insurance premium from a particular insurer and a non-price 'cost' pertaining to the affinity between the customer's preferences and the insurer's characteristics. (This is explained in greater detail later.) Customers select the lowest-cost insurer, based on the 'total cost', and the whole market is cleared, balancing supply and demand. Insurers collect premiums after being selected by their customers at year-start. During the year, customers' total claims are randomly generated and paid by insurers at year-end (deductibles are ignored, or assumed to be identical for all customers). Insurers update their underwriting results after paying claims. Based on performance, insurers decide what their competitive price per unit of risk will be in the next period. The market process then recommences at the beginning of the next period.

Pricing. It is a standard result in neoclassical price theory that a firm's profits are maximized when marginal cost (MC) equals marginal revenue. This occurs when the firm charges the price P^* given by

$$P^* \;=\; MC \times (1 + \epsilon^{-1})^{-1} \tag{1}$$

where ϵ is the price elasticity of demand. See for example Hirschey and Pappas (1996, p. 637) or Petersen and Lewis (1999, p. 429).

Equation (1) is an example of cost-plus or mark-up pricing. Price is formulated as a function of cost which is marked-up through the term $(1+\varepsilon^{-1})^{-1}$. If a market is competitive and its demand elasticity is high, then the price-cost margin $(P^* - MC)/MC$ and mark-up P^*/MC will be lower. In a perfectly competitive market, with perfect elasticity of demand, there is no mark-up and firms charge a price equal to the marginal cost and make no economic profit.

We assume in our model that insurers are boundedly rational when pricing their policies, and use cost-based or mark-up pricing. More specifically, they price their policies using a modified form of equation (1). First, in the absence of information on incremental cost and revenue, insurers use the average cost of a policy. Long-run average cost and marginal cost are often not too different (Petersen and Lewis, 1999, p. 429). Much of the cost is variable rather than fixed (Feldblum, 2001). Second, the average cost is based on established actuarial premium principles (Kaas et al., 2008). In equation (1), MC is replaced by $\widetilde{P}_{it} + \alpha F_{it}$, where $\widetilde{P}_{it}$ is the pure premium based on the expected claim cost and αF_{it} is a risk loading or safety loading ($\alpha > 0$ is a loading factor and F_{it} is the standard deviation of claims experienced by insurer i by time t). Third, we define the log-mark-up m_{it} employed by insurer i at time t through the following equation:

$$P_{it} \; = \; \left(\widetilde{P}_{it} + \alpha F_{it}\right) e^{m_{it}}. \tag{2}$$

To evaluate the optimal mark-up, the management of an insurance company must estimate price-elasticity of demand. This varies (a) at different price points, (b) as competitors enter or exit particular segments of the market, (c) as the insurer moves in and out of certain segments, (d) at different stages of the underwriting cycle. Insurers cannot therefore estimate their price elasticity of demand precisely (Warthen and Somner, 1996). Note that individual insurers will experience different price sensitivity in different segments of the market and at different times. For the market as a whole, demand for automobile insurance is relatively inelastic relative to price; for an individual insurer, demand is very elastic (Feldblum, 2001).

We assume that insurer i calculates a crude elasticity of demand at time t using an arc-elasticity over the previous two periods (Hirschey and Pappas, 1996, p. 639):

$$\widehat{\epsilon}_{it} \; = \; \frac{(Q_{i,t-1} - Q_{i,t-2})/(Q_{i,t-1} + Q_{i,t-2})}{(P_{i,t-1} - P_{i,t-2})/(P_{i,t-1} + P_{i,t-2})}, \tag{3}$$

where $Q_{i,t}$ denotes the number of insurance policies sold by insurer i at time t and $P_{i,t}$ denotes the price of these policies. (The above approximates the point elasticity of demand $\epsilon = \frac{\partial Q/Q}{\partial P/P}$ where Q is quantity demanded and P is price.)

Comparing equations (1) and (2), and using equation (3), a first-order approximation suggests a crude estimate of the mark-up as follows:

$$\widehat{m}_{it} \;\approx\; -\frac{1}{\widehat{\epsilon}_{it}} \;=\; -\frac{(P_{i,t-1} - P_{i,t-2})(Q_{i,t-1} + Q_{i,t-2})}{(Q_{i,t-1} - Q_{i,t-2})(P_{i,t-1} + P_{i,t-2})} \tag{4}$$

This approximation is based on $-\widehat{\epsilon_{it}}^{-1}$ being typically small in a highly competitive market where individual insurers face a gently sloping, and nearly horizontal, demand curve. Finally, insurer i estimates the optimal mark-up by updating its previous estimate using a weighted average:[1]

$$m_{it} \;=\; \beta\widehat{m}_{it} \;+\; (1-\beta)m_{i,t-1}, \tag{5}$$

where $0 < \beta \leq 1$. It is noteworthy that exponential weighted moving averages such as in equation (5) occur in macroeconomic models of the business cycle with variable mark-ups and sticky prices (see e.g. Rotemberg and Woodford, 1999).

Location and distance. To operationalize monopolistic competition, we employ a location model (Salop, 1979; Schlesinger and Schulenburg, 1991). A key difference between monopolistically and perfectly competitive markets is that products are heterogeneous in the former, because strategic competition between firms leads to differentiation in terms of non-price characteristics, such as insurer's reliability and trustworthiness (Wells and Stafford, 1995), reputation and branding, marketing and advertising methods, target demographics etc. In a location model, location can represent physical or geographic location (Degryse and Ongena, 2005, e.g.). More generally, one may assume an abstract location space, with a pre-defined topology such as a finite line or a circle, regarded as a "preference space for product characteristics" (Schlesinger and Schulenburg, 1991). Location represents a consumer's preferences as to non-price attributes of the firm, and distance measures the affinity between the customer's non-price preferences and the firm's attributes.

[1] In the (rare) circumstances where $Q_{i,t-1} = Q_{i,t-2}$, which would lead to an infinite value of $\widehat{m}_{it}$ in equation (4), one may calculate the arc-elasticity using values at time $t-3$, but this would lead to outdated estimates based on different market conditions and possibly distant price points. Instead, we assume that the insurer does not update his estimate in equation (5) and sets $\widehat{m}_{it} = m_{i,t-1}$.

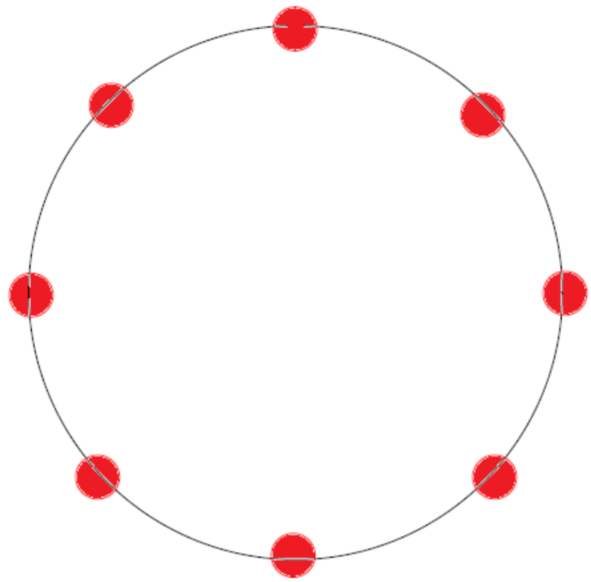


Figure 1: An economic location model of the automobile insurance market: insurers (large dots) are evenly distributed along an abstract circular space of product attributes (such as insurer's perceived reliability, size, marketing methods etc.), with a large number of insureds distributed uniformly along the circle. The closer a customer is to an insurer, the greater his affinity for the insurer and its product.

In our model, we use a circular location space, as shown in Figure 1. This originates from the 'circular city' model of Salop (1979), but has been used extensively, for example by Ladley (2013) in his inter-bank lending model to define relationships between banks and households and study contagion effects. There are a large number of insurance consumers uniformly distributed along the large circle in Figure 1. Insurers, shown as large dots on Figure 1, are also evenly located on the circular space. The closer a customer is to an insurer, the greater the preference the customer has for the insurer's product attributes.

Customer's total cost function. The circumference of the circle in Figure 1 is normalized to one. Define Δ_{ij} as the shortest distance between insurer i and customer j along the circumference of the circle. Then, customer j has the following total cost function of purchasing a policy from insurer i at the start of of period $(t, t+1)$:

$$TC_{ijt} = P_{it} + \gamma\Delta_{ij}, \tag{6}$$

Decision rules based on more elaborate utility functions, such as in Schlesinger and Schulenburg (1991), could be used, but equation (6) has the virtue of simplicity. In the above, γ is a cost per unit of distance, similar to utility loss per unit of distance in Schlesinger and Schulenburg (1991), i.e. it signifies a reduction in utility from having to buy a policy which does not satisfy the consumer's ideal product characteristics.

Capital and claims process. Every customer ranks all insurers from the lowest total cost to the highest. The customer then chooses the insurer with the lowest total cost, from the customer's point of view, unless this insurer has reached full capacity, in which case the customer chooses the insurer with the next lower total cost etc. An insurer exhausts its capacity if it uses all of its existing capital to support its insurance business. Once full capacity is reached, the insurer stops taking other customers. If an insurer has more interested customers than it can support, the insurer selects the customers at random, so that there is no selection bias. An insurer's total capacity in each time period depends on its existing level of capital which defines the maximum gross premiums that an insurer can take in each time period. During the policy year, a stochastic claim loss experience is generated. Policyholders' claims are independent and identically distributed. Claim frequency is governed by a Bernoulli distribution with parameter b, and claim severity is modeled using a Gamma distribution with mean μ_G and standard deviation σ_G. For the sake of brevity, we omit other minor details here and refer the reader to Zhou (2014).

Re-pricing. At the end of each time period, insurers collect information from their own experience and from the market to update their prices. They use information such as the mean and standard deviation of their own total portfolio loss to update their safety loading in equation (2). The quantity of insurance sold is used to update their mark-up adjustment in equation (4).

They also calculate a credibility-weighted estimate of the pure premium $\widetilde{P}_{it}$, in equation (2), based on their own claim experience and the market-collected average claim. Actuarial costing methods are well-documented, and here we follow Kaas et al. (2008, p.

203–227). The pure premium is calculated by each insurer according to:

$$\widetilde{P}_{it} \; = \; z\bar{X}_{it} \, + \, (1-z)\lambda\mu, \tag{7}$$

where $\overline{X}_{it}$ is the past weighted average claims of insurer i at time t:

$$\overline{X}_{it} \; = \; wX_{i,t-1} \, + \, (1-w)\overline{X}_{i,t-1}. \tag{8}$$

In equation (7), λ and μ represent the average claim frequency per unit risk exposure and the average claim size respectively, as collected across all insurers during the simulation. (This replicates the pooling of claims information through insurance rating bureaus or advisory organizations such as the Insurance Services Office in the US or the Claims and Underwriting Exchange in the UK.) Thus, $z \in [0, 1]$ is a credibility factor which weights the insurer's own claims experience against the market experience as a whole. In equation (8), $X_{i,t-1}$ is the realized claim against insurer i at time $t-1$. The weight $w \in [0, 1]$ determines the importance of the experience of the previous period against the past claim history, given that more recent data are usually judged by insurers to be more relevant to the calculation of current risk premium.

4. Data, Model Estimation, and Validation

We use actual annual data on loss ratios and combined loss ratios from the UK automobile insurance market (or "UK motor insurance market") to calibrate and validate our model. The data is collected from the Association of British Insurers (ABI) database between year 1983 and 2011. A common problem to all studies on the underwriting cycle is that the annual data series that are available are short, especially relative to the cycle lengths of 5–10 years which are typically quoted. The small data length means that cycle length estimates are subject to large errors (Boyer, Jacquier and Van Norden, 2012; Boyer and Owadally, 2015). Even if longer data were available, structural changes in the market, for example because of regulation, technology, new product development and distribution methods etc. may render the earlier data points irrelevant. One motivation for the use of

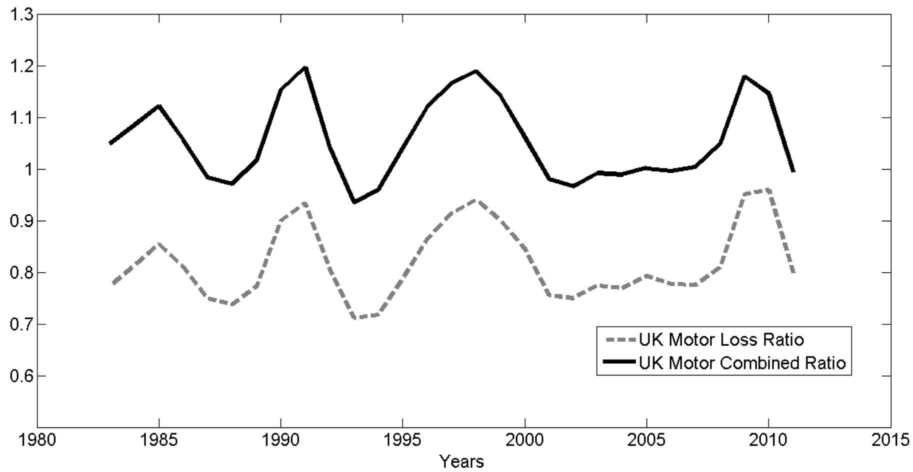
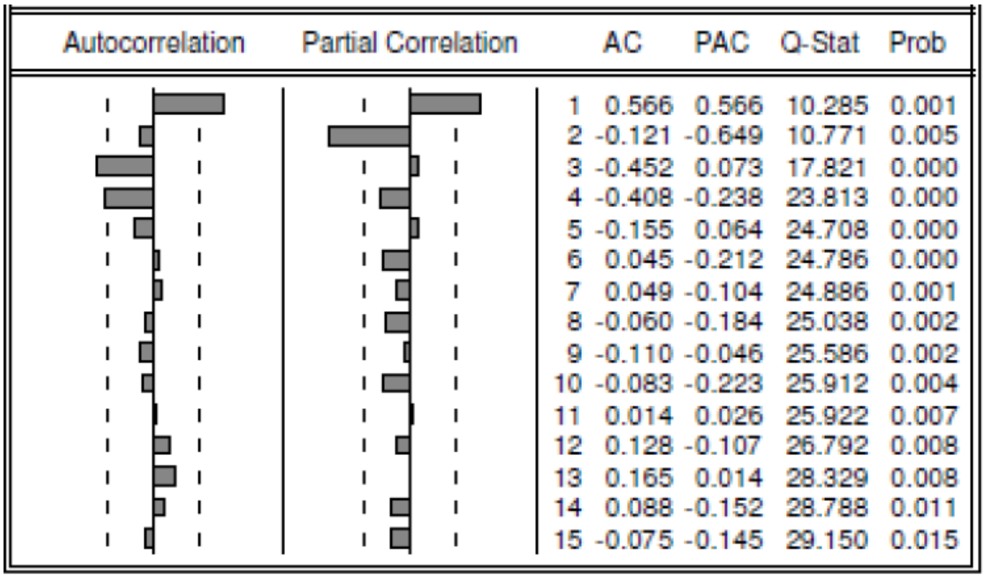


Figure 2: Loss ratios and combined loss ratios for the UK automobile (or 'motor') insurance market. Source: Association of British Insurers.

agent-based models is that their rich real-world structure can in part mitigate such data restrictions.

Figure 2 shows both UK motor insurance market loss ratios and combined ratios from 1983 to 2011. The two series of ratios follow a similar, apparently cyclic pattern. The main difference between them is the expense, which appears to be stable over time as a proportion of premium, at about 20–25 percent. In what follows, we use the combined loss ratio data. The autocorrelations and partial autocorrelations, over different lags, of the data are shown in Figure 3. Note the gently undulating but rapidly declining values of the autocorrelations as lag increases, indicating possible stationary cycles.

The parameters and parameter values in our model are summarized in Table 1. We describe the parameterization procedure that we follow in detail in the Appendix, and summarize this briefly here. Numerical experiments show that the three key parameters which govern the dynamics of the market are the first three parameters (α, β, γ) in Table 1. Unless they take unreasonable extreme values, the other parameters have a much less material effect. The values for these parameters are chosen to achieve a realistic representation of the market, while avoiding an unduly long simulation time. In particular, M, N and T in Table 1 are chosen for reasonableness, the claims process parameters μ_G,



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.566	0.566	10.285	0.001
		2	-0.121	-0.649	10.771	0.005
		3	-0.452	0.073	17.821	0.000
		4	-0.408	-0.238	23.813	0.000
		5	-0.155	0.064	24.708	0.000
		6	0.045	-0.212	24.786	0.000
		7	0.049	-0.104	24.886	0.001
		8	-0.060	-0.184	25.038	0.002
		9	-0.110	-0.046	25.586	0.002
		10	-0.083	-0.223	25.912	0.004
		11	0.014	0.026	25.922	0.007
		12	0.128	-0.107	26.792	0.008
		13	0.165	0.014	28.329	0.008
		14	0.088	-0.152	28.788	0.011
		15	-0.075	-0.145	29.150	0.015

Figure 3: Autocorrelations and partial autocorrelations of loss ratios for the UK automobile insurance market

σ_G and b are chosen to be consistent with previous simulation studies (Taylor, 2008), and the actuarial premium calculation parameters w and z correspond to typical values used by practitioners in European insurance markets (Kaas et al., 2008; Booth et al., 2005).

The bulk of the estimation work concerns the three key model parameters α, β and γ. Estimation of these parameters proceeds by a version of the method of moments, using repeated Monte-Carlo simulations of the agent-based model on a grid of values, with successive grid search and refinement. The grid search is carried out to match the mean, standard deviation and lag-1 autocorrelation of the simulated loss ratios to the corresponding statistics from the actual loss ratios. Figure 4 shows an excerpt of the grid with β and γ only. The grid is then refined around the best triplet of parameter values, simulations are performed again, and a further grid search is carried out. The process is repeated until a reasonable degree of precision for the optimal estimates of α, β and γ is achieved. The Appendix describes the procedure in greater detail.

Two remarks, about the stochastic simulations used to estimate the moments of the simulated loss ratios, are in order. First, the simulated loss ratios appear to be stationary and ergodic (this is discussed further below). This means that the statistics for the loss

Parameter	Value	Definition
α	0.001	Risk loading factor in insurers' premium calculation, eq. (2)
β	0.3	Weight in insurers' mark-up calculation, eq. (5)
γ	0.05	Weight in customers' total cost calculation, eq. (6)
M	1000	Number of customers
N	20	Number of insurers
T	1000	Time horizon of simulation
μ_G	100	Mean of Gamma-distributed i.i.d. claims
σ_G	10	Standard deviation of Gamma-distributed i.i.d. claims
b	1	Parameter of claim frequency distribution
w	0.2	Weight in estimation of insurer's average claim size, eq. (8)
z	0.2	Credibility factor in pure risk premium, eq. (7)

Table 1: Parameters and parameter values in the agent-based model.

Standard Deviation					
Beta	0	0.3	0.5	0.7	1
Gamma					
0	0.0038	0.0038	0.0038	0.0038	0.0038
0.03	0.0032	0.0714	0.3233	0.3919	0.5543
0.05	0.0032	0.0735	0.1758	0.2390	0.8876
0.07	0.0032	0.0781	0.1584	0.2110	0.2793
0.1	0.0032	0.0980	0.1682	0.1975	0.2587

Rho1					
Beta	0	0.3	0.5	0.7	1
Gamma					
0	0.0178	0.0178	0.0178	0.0178	0.0178
0.03	-0.0208	0.289	0.2088	0.061	-0.109
0.05	-0.0208	0.3914	0.1217	-0.0295	-0.0178
0.07	-0.0208	0.403	0.2128	0.0313	-0.1015
0.1	-0.0208	0.5474	0.0865	-0.0444	-0.0144

Figure 4: Standard deviation and lag-1 autocorrelation (Rho1) of simulated loss ratios for different values of β and γ parameters.

ratios can be calculated from one long simulated sample path (1000 periods), rather than an ensemble of sample paths. An initial run-in of 100 periods is discarded to avoid transient results at initialization. Second, we repeat the simulations with different time horizons to verify that our estimates are stable. We also use different randomizations of the stochastic claims made by policyholders and again verify our estimates. Of course, to avoid sampling error, the same pseudo-random sample of claims is generated when carrying out the grid search in the α-β-γ parameter space.

Having parameterized the model, we proceed with a few validation tests. First, we investigate the second-order stationarity of loss ratios, both actual loss ratios in the data and simulated loss ratios from the model. De-trending was not required. The Augmented Dickey-Fuller (ADF) test comfortably rejects the null hypothesis of the presence of a unit root at 5% in both the actual and simulated loss ratio time series (actual data: ADF test statistic = -6.075, 1% critical value = -3.696; simulated: ADF test statistic = -3.504, 5% critical value = 2.975). The Phillips-Perron (PP) test rejects a unit root at 5% for the simulated time series, but rejects it only at 10% for the actual loss ratio data (actual data: PP test statistic = -2.611, 10% critical value = -2.624; simulated: PP test statistic = -3.142, 5% critical value = -2.971).

Similar test results and decaying correlograms are obtained on other simulated sample paths, so we conclude that the simulated loss ratios are stationary. We do not formally test for joint ergodicity and stationarity, but numerical work on several simulated sample paths indicate that long-run means and standard deviations along sample paths are stable and approximately equal to ensemble means and standard deviations.

Given stationarity and ergodicity, it is sensible to consider the distribution of loss ratios. The empirical cumulative distribution functions of the actual loss ratios and one sample path of simulated loss ratios are shown in Figure 5. If our model has a good fit to the data, then the distribution from a simulated sample path should be close to the frequency distribution of loss ratios from the actual UK automobile insurance market. We use a two-sample Kolmogorov-Smirnov (KS) test to compare the distribution of the simulated loss ratios with the distribution of the actual market loss ratios. (The alternative Pearson

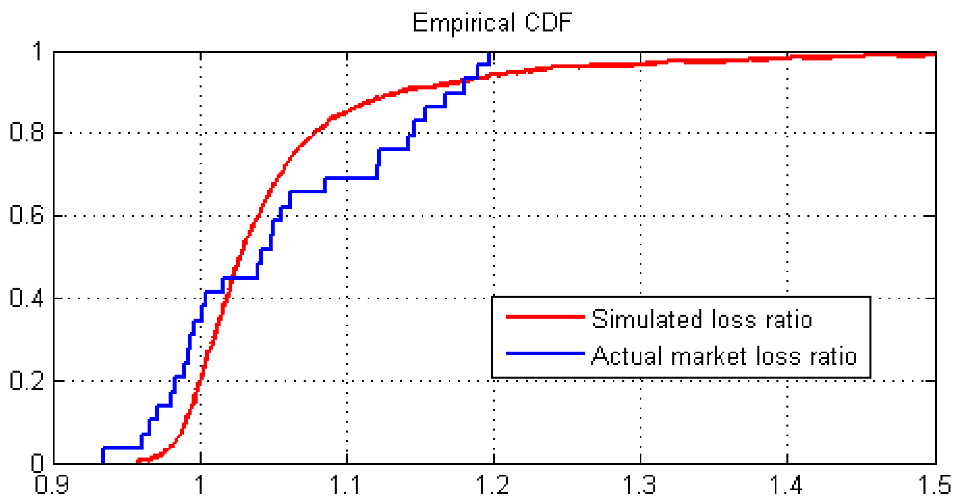


Figure 5: Empirical cumulative distribution functions (CDF) of simulated loss ratios (smooth line) and actual loss ratios (stepped line) from the UK automobile insurance market.

χ^2 goodness-of-fit test could also be used, but it is less powerful for small sample sizes, so we prefer the KS test.) At the 5% level, the KS test cannot reject the null hypothesis that the simulated loss ratios and the actual market loss ratios come from the same continuous probability distribution (p-value = 0.1997). This confirms that our model has a close fit to the actual insurance market.

We also provide a simple economic validation for our model by considering the distribution of insurance firm size. The real UK property-casualty ("non-life") insurance market exhibits a typical heavy-tailed or "power-law" distribution, i.e. one where a small number of large insurers have a large market share. As shown in Table 2, almost 90% of market share is held by the largest 10 insurers. Does our agent-based model lead to a market with a power-law distribution of firm size?

For repeated Monte-Carlo simulations of our model, where all insurers start off with the same amount of capital, we collect the insurers' final capital accumulation at a fixed time horizon of 1000 years. The distribution of the insurers' capital is shown in the left-hand panel of Figure 6. The distribution declines slowly and there are a few very large insurers dominating the simulated market, much as in the real market illustrated in Table 2. The

Business Class	Market share 2010	Market share 2011
Personal line top 5 insurers	63.7%	63.0%
Personal line top 10 insurers	85.3%	82.9%
Personal line top 20 insurers	98.5%	97.0%
Commercial line top 5 insurers	67.8%	65.7%
Commercial line top 10 insurers	90.4%	87.1%
Commercial line top 20 insurers	98.8%	97.2%
Whole industry top 5 insurers	66%	64%
Whole industry top 10 insurers	88%	85%
Whole industry top 20 insurers	99%	97%

Table 2: Property-Casualty (non-life) insurance market share in the UK by size of insurer. Source: Association of British Insurers.

right-hand panel of Figure 6 confirms that the distribution of insurer size does indeed have a heavy tail: the log-log plot declines approximately as a straight line. This shows that our model behaves much like the actual UK insurance market.

5. Simulation Results and Discussion

A sample path of simulated loss ratios from our agent-based model is shown in Figure 7. Casual inspection suggests that the simulated loss ratio appears to exhibit cycles. The simulated loss ratios may be compared to the loss ratios in the actual UK automobile insurance market in Figure 2. The actual and simulated cycles have a trough-to-crest depth that are of a similar size (the vertical scales of Figures 2 and 7 are identical).

Much of the literature on insurance cycles employs a second-order autoregression to determine the length of cycles. See, for example, Cummins and Outreville (1987), Lamm-Tennant and Weiss (1997) and Harrington, Niehaus and Yu (2014). This is challenged by Boyer, Jacquier and Van Norden (2012) and Boyer and Owadally (2015) because the sparseness of the data renders both the identification of cycles and estimation of cycle

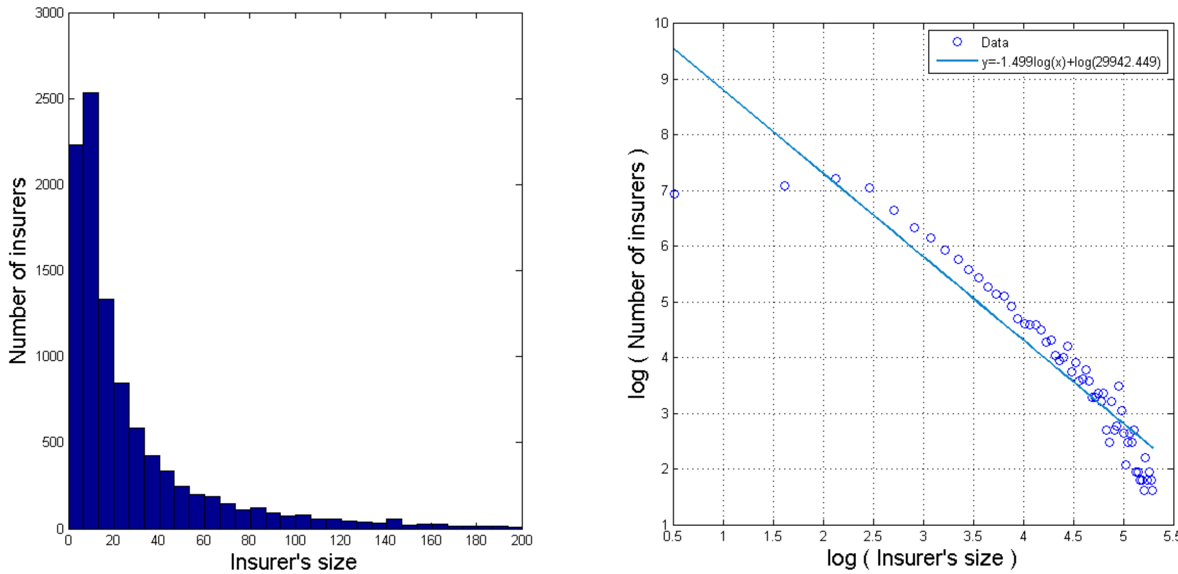


Figure 6: Left-hand panel: histogram of insurers' size (by capital held) in simulated agent-based model. Right-hand panel: the right-hand tail of the distribution of insurer size in the simulated model declines as a power law, illustrated here by the nearly linear declining log-log plot.

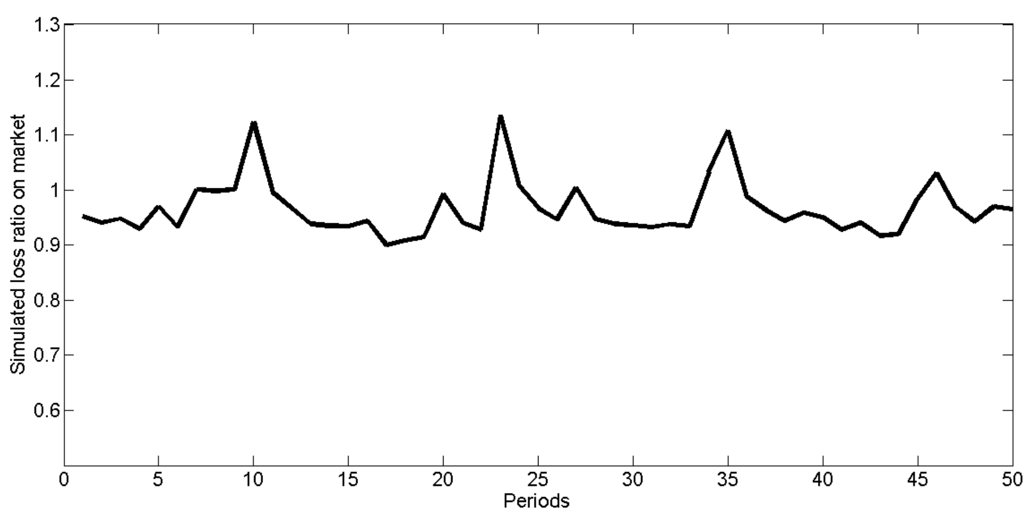


Figure 7: Sample path of simulated loss ratio in agent-based model of UK automobile insurance market.

Loss ratios	Φ_0	Φ_1	Φ_2	Adj. R^2	Period (years)
Actual	1.0489	1.0873	-0.8289	0.698	6.75
	(89.76)	(7.790)	(-5.794)		
Simulated	1.0297	0.5468	-0.2710	0.183	6.17
	(34.98)	(2.789)	(-1.368)		

Table 3: Parameterization of AR(2) model, $L_t = \Phi_0 + \Phi_1 L_{t-1} + \Phi_2 L_{t-2} + \epsilon_t$, where L_t denotes loss ratio at time t, ϵ_t denotes the residual. t-statistics are shown in parentheses. The cycle period is also shown.

periods difficult. Our agent-based model, however, can generate long simulated time series, so their criticism is not directly applicable.

Stationarity tests for both actual and simulated loss ratios was discussed earlier. Fitting an AR(2) model to both actual data and simulated loss ratios yields the parameter values in Table 3. The parameter values for the actual loss ratios on the UK automobile insurance market are comparable to those reported for example by Harrington, Niehaus and Yu (2014) on other insurance data, but the parameter values for the simulated loss ratios are lower in magnitude than those typically reported, albeit of a similar sign. It is well-known that the autoregressive parameters Φ_1 and Φ_2, as reported in Table 3, must satisfy $\Phi_1^2 + 4\Phi_2 < 0$ for cycles to exist. (Conditions for stationary cycles are discussed further in Boyer and Owadally (2015).) The inequality is satisfied by the AR(2) models fitted to both the actual loss ratio data and the simulated loss ratios from the agent-based model, indicating the presence of cycles in both cases.

A cycle length can be calculated using the formula given by Cummins and Outreville (1987, eq. (15)) among others. The last column of Table 3 shows that the estimated cycle lengths from the actual UK automobile insurance market and the simulated market of the agent-based model are comparable and are of the order of 6–7 years.

Looking at the peak-to-peak distances in the time plot of Figure 2 for actual market loss ratios, the estimated cycle length of 6.75 years does seem to be reasonable. However, the peak-to-peak distances in the time plot of Figure 7 suggest a cycle length longer than 6.17

	$p = 0$	$p = 1$	$p = 2$	$p = 3$
$q = 0$	–	-2.500	**-3.270**	-3.238
$q = 1$	-3.652	**-3.771**	-3.331	–
$q = 2$	-3.639	-3.684	-3.417	–
$q = 3$	-3.291	–	–	–

Table 4: Akaike Information Criterion (AIC) values for some ARMA(p,q) models fitted to the actual loss ratios on the UK automobile insurance market. The AIC values for the AR(2) and ARMA(1,1) models are highlighted: the ARMA(1,1) model has the best fit to the data among the models considered here.

years in the agent-based model. Furthermore, the adjusted R^2 for the AR(2) model fitted to the simulated loss ratios is low at 18% (Table 3), so the AR(2) is a poor fit to loss ratios in the agent-based model. The AR(2) coefficients, reported in Table 3 as Φ_1 and Φ_2, are different between the actual and simulated loss ratios, although they are of the same sign. This underscores the point made by Boyer, Jacquier and Van Norden (2012) and Boyer and Owadally (2015) that the cycle length estimator in an AR(2) model is non-linear in the AR(2) coefficients, so that different stationary AR(2) models can nevertheless produce similar cycle period estimates.

In fact, the AR(2) model does not provide the best fit to the actual loss ratios on the UK automobile insurance market according to the Akaike Information Criterion (AIC). Table 4 shows that a number of other ARMA models provide a better fit than AR(2), with ARMA(1,1) appearing to be the best. Conclusions based on the AR(2) model should therefore be treated with caution, which gives further credence to the arguments of Boyer, Jacquier and Van Norden (2012) and Boyer and Owadally (2015).

Nevertheless, it is difficult to deny the appearance of cycles in both the actual UK automobile insurance market (Figure 2) and our simulated market (Figure 7). In particular, the fact that the agent-based model appears to be cyclical, despite the absence of any obvious second-order dynamic mechanism, is of great interest. One way to investigate the presence of cycles is by spectral analysis (Doherty and Kang, 1988; Venezian, 2006; Lazar

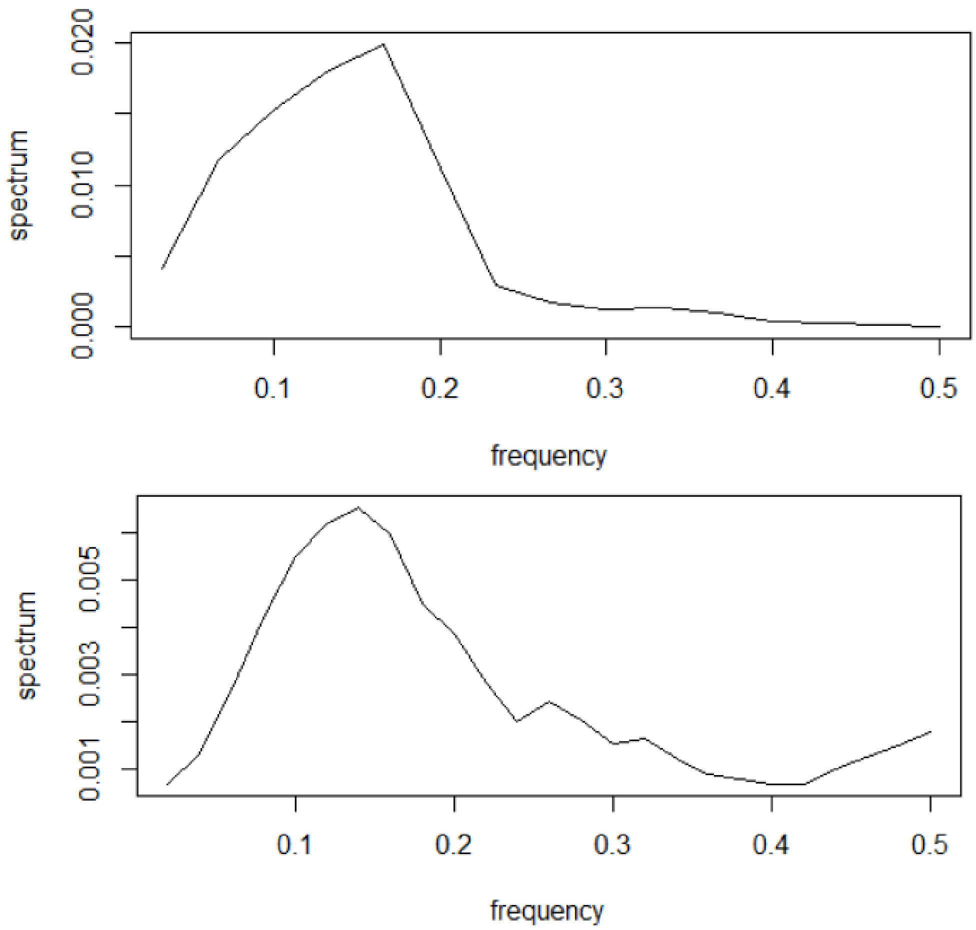


Figure 8: Smoothed periodograms (bandwidth = 0.025) of actual loss ratios on UK automobile insurance market (top) and simulated loss ratios from agent-based model (below).

and Denuit, 2012). Figure 8 depicts the spectra of actual and simulated loss ratios. They both exhibit a peak at an annual frequency of around 0.15, which corresponds to a cycle period of about 6.7 years.

An obvious question raised from our agent-based model is whether imperfect (monopolistic) competition and insurers' bounded rationality, the two key features of our model, are a cause of underwriting cycles. We cannot answer this question directly but we can experiment with the model to see whether we obtain a purely random sequence of loss ratios with the presence or absence of these features. We carry out four experiments by repeating the Monte-Carlo simulations and varying only the β and γ parameters as shown

Experiment	β	γ	p-values	
			Ljung-Box test	Runs test
1	1	0	0.116	0.919
2	0.3	0	0.116	0.919
3	1	0.05	0.994	0.016
4	0.3	0.05	0.036	0.000

Table 5: Four experiments with various values of β and γ, and p-values of tests for autocorrelations and randomness in one sample path of simulated loss ratios.

in Table 5.

In experiment 1 of Table 5, there is neither bounded rationality nor monopolistic competition in the model. $\beta = 1$ means that insurers update their price mark-up on the actuarial cost without reference to previous years' prices. They seek to maximize profits by responding directly to the change in demand over the last period as their prices changed. This is not identical to rational pricing, but is as close as possible to pricing according to equation (1). $\gamma = 0$ means that customers place no weight on non-price differences between insurers, and they respond only to the prices that are offered by these insurers, so that all insurance products are interchangeable and the market becomes perfectly competitive. In experiment 2 of Table 5, insurers are boundedly rational, with β set at the estimated value previously obtained, but there is perfect competition ($\gamma = 0$). In experiment 3, bounded rationality is absent ($\beta = 1$), but the market is monopolistic with $\gamma = 0.05$ as previously estimated. Experiment 4 of Table 5 is merely the model as estimated and validated on UK automobile insurance data.

For each of the experiments above, we run the Ljung-Box portmanteau test and the runs test on one sample path of loss ratios. The Ljung-Box test statistic is χ^2-distributed with degrees of freedom equal to the number of lags (20) being tested. (The test is not being carried out on residuals and no parameter was estimated, so the degrees of freedom are not reduced by the number of estimated parameters.) The fourth column of Table 5 shows that, at the 5% significance level, we fail to reject the null hypothesis that the loss ratios are an uncorrelated sequence in experiments 1–3. Unsurprisingly, we do reject the null hypothesis in experiment 4, since the agent-based model was parameterized to actual market loss ratios which are autocorrelated. This indicates that *both* bounded rationality *and* monopolistic competition must be present for the loss ratios to be serially correlated. Further simulations and testing suggest that this result is robust to reasonable changes in β and γ ($0 < \beta < 1$, $\gamma > 0$).

The runs test is somewhat less clear-cut. The test is based on the number of runs of consecutive values above or below the mean loss ratio. The p-values in the fifth column of Table 5 are greater than 5% for experiments 1 and 2, so we cannot reject the null hypothesis that the loss ratios are a serially independent sequence, in these two cases. We do reject the null hypothesis, with very high significance, in the fourth baseline case when the agent-based model is fitted to actual loss ratio data. In the third experiment, when insurers are rational but the market is monopolistically competitive, we also reject the null hypothesis that the loss ratios form a purely random sequence. This is not inconsistent with the Ljung-Box test, however, since the loss ratios may be serially dependent but uncorrelated (assuming that they have finite moments).

Although the statistical tests above are limited, they do suggest that it is the *combination* of monopolistic competition *and* bounded rationality that trigger cyclical effects on the insurance market. Just one of these effects by itself does not generate complexity and does not allow for the emergence of cycles. In other words, small departures from the idealized conditions of *both* perfectly competitive insurance markets *and* fully rational insurance pricing may be a contributory factor to underwriting cycles.

Of course, this result is dependent on some of the key assumptions of our model. For example, the choice of a circle (Figure 1) to represent the non-price preferences of insurance consumers was made arbitrarily. We have investigated other topologies but have found no notable difference in our conclusions (Zhou, 2014). Our conclusions are also robust to the form of the total cost function in equation (6). This is in line with most of the literature on agent-based models with location models. For example, in the inter-bank lending model of

Ladley (2013), location determines a bank's attractiveness when depositors select a bank to place their funds. Ladley (2013) uses a linear distance to model costs, but finds that alternative functions generate few qualitative differences.

6. Conclusion

In this paper, we propose a mechanism based on imperfect competition and insurers' bounded rationality for the existence of insurance cycles. Insurers cannot perfectly read their customers' preferences and practice cost-based pricing in reality; we interpret this as a form of boundedly rational pricing behavior. Insurers are also heterogeneous in terms of their non-price product attributes such as perceived reliability, reputation, marketing methods etc. Customers are concerned with price as well as non-price product characteristics, leading to a monopolistically competitive market. The dynamics of such an imperfectly competitive market appears to causes mispricing to propagate in ups and downs, thereby giving rise to cycles in the market as a whole.

The model is implemented as an agent-based model by simulating each insurer and insurance consumer, who both follow simple behavioral rules, and by capturing the interaction between all agents. An abstract circular location space is used to operationalize monopolistic competition on the insurance market. The model is then estimated on actual UK automobile insurance market data through repeated Monte-Carlo simulations and grid search with refinement. The distribution of simulated market loss ratios has a good fit to the actual distribution of loss ratios on the UK automobile insurance market. The distribution of insurers' size in the simulated agent-based market is found to be right-skewed and heavy-tailed, similar to the actual market which is dominated by a few very large insurers.

No exogenous shocks such as catastrophic loss shocks or interest rate shocks are allowed to occur in our model, and there is no binding solvency constraint. Cycles appear to emerge endogenously, as confirmed by peaks in spectra as well as a second-order autoregression on simulated sample paths of loss ratios in the agent-based model. Simulation experiments suggest that both imperfect competition and bounded rationality are required to generate complex market behavior and give rise to cycles. This suggests that small departures away from perfect competition and fully rational pricing are a possible contributory cause to underwriting cycles on property-casualty insurance markets.

Appendix: Parameterization Procedure

The parameters and parameter values in our model are summarized in Table 1. Numerical experiments show that the three key parameters which govern the dynamics of the market are the first three parameters in Table 1. Unless they take unreasonable extreme values, the other parameters have a much less material effect. The estimation of the first three parameters is discussed below, but we first explain how the remaining parameter values are chosen.

Parameter values for M, N and T are chosen for their reasonableness. We start with a sensible range for each, $N \in [2, 100]$, $M \in [100, 1000000]$, $T \in [50, 10000]$, and continually narrow down the range until no material difference appears, by visual inspection, in the simulation results. We then seek to balance the simulation time and a realistic representation of the market. The parameters μ_G, σ_G, and b, concerning the claims process, appear to make little difference except in the scale and volatility of loss ratios. Recall that, by assumption, the claims process in our model excludes one-off catastrophic claims. For these parameters, we use values that are consistent with the existing literature, e.g. the simulation study of Taylor (2008). Finally, for parameters w and z, which relate to actuarial rate-making, we adopt common rules of thumb used by insurance practitioners (Booth et al., 2005). For example, 20/80 weights are used in the credibility estimate of the pure risk premium (Kaas et al., 2008). A sensitivity analysis shows that changes in these parameters have no significant qualitative impact on our simulation results.

The three key parameters in our model are α, β and γ. Estimation of these parameters proceeds by a version of the method of moments, using repeated Monte-Carlo simulations of the agent-based model for different values of (α, β, γ). Because loss ratios are autocorrelated, we would like to solve for (α, β, γ) by equating the mean $\overline{\mu}$, standard deviation $\overline{\sigma}$ and lag-1 autocorrelation $\overline{\rho}$ of the simulated loss ratios to the corresponding sample statistics $(\widehat{\mu}, \widehat{\sigma}, \widehat{\rho})$ of the actual market loss ratios.

In theory, we could use a grid search procedure whereby we try a large number of combinations of values for (α, β, γ), carry out a large number of stochastic simulations, estimate $\overline{\mu}$, $\overline{\sigma}$ and $\overline{\rho}$ for the simulated loss ratios, minimize a weighted penalty function such as $\|\overline{\mu} - \widehat{\mu}\| + w_1\|\overline{\sigma} - \widehat{\sigma}\| + w_2\|\overline{\rho} - \widehat{\rho}\|$, where w_1 and w_2 are suitably chosen weights. We may then refine the grid around the penalty-minimizing triplet (α, β, γ), and start the grid search again, etc.

In practice, we observe from our simulations that the mean loss ratio $\overline{\mu}$ on the simulated market is governed by α, and that β and γ have a negligible effect on $\overline{\mu}$. Furthermore, α has little effect on the higher moments of the simulated loss ratio. We can see that this is true by construction in our model if we assume that the mark-up (or elasticity of demand) is serially independent of aggregate claims (from equations (2), (4) and (5)). This means that we can run stochastic simulations and choose the value of α such that $\overline{\mu} = \widehat{\mu}$. We actually choose to set α such that the mean simulated loss ratio $\overline{\mu} = 100\%$. This ensures that the long-run state of the simulated market remains competitive and stable. If $\overline{\mu} > 1$, then most insurers become insolvent in the long run. If $\overline{\mu} < 1$, then profitability is such that new firms would enter the market, something which is precluded in our model. (Recall also that expenses and investment return are ignored.)

We then use a brute-force grid search procedure with refinement to estimate β and γ: there are several pairs of values of (β, γ) which satisfy the standard deviation matching equality $\overline{\sigma} = \widehat{\sigma}$. A surface is obtained when plotting $\overline{\sigma}$ against β and γ, and there is a line of values of (β, γ) corresponding to the sample standard deviation $\widehat{\sigma}$. Likewise, there is no unique pair of values of (β, γ) which satisfies $\overline{\rho} = \widehat{\rho}$. One possible strategy is to minimize a weighted sum of $\|\overline{\sigma} - \widehat{\sigma}\|$ and $\|\overline{\rho} - \widehat{\rho}\|$, but the choice of the weight is arbitrary. For reasons of practicality, we give more weight to standard deviation matching and choose (β, γ) such that the standard deviation of loss ratios in the simulated model approximates that of the real market data, $\overline{\sigma} \approx \widehat{\sigma}$, and also minimizes $|\overline{\rho} - \widehat{\rho}|$.

The grid search is done by repeated Monte-Carlo simulations over a grid of values of β and γ, with statistics from the simulated loss ratios being calculated each time. Figure 4 shows an excerpt of the grid. The grid search is then manually refined in the vicinity of the best estimate of (β, γ), and the search started again, until a suitable level of precision is achieved. The parameter estimates for α, β and γ appear in Table 1.

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