Space races: settling the universe *fast*

Anders Sandberg
Future of Humanity Institute,
Oxford Martin School,
University of Oxford

January 24, 2018

Abstract

This report examines the issue of the resource demands and constraints for very fast large-scale settlement by technologically mature civilizations. I derive various bounds due to the energy and matter requirements for relativistic probes, and compare with bounds due to the need to avoid collisions with interstellar dust. When two groups of the same species race for the universe, the group with the biggest resources completely preempts the other group under conditions of transparency. Expanding nearby at a lower speed in order to gain resources to expand far at a high speed is effective. If alien competitors are expected this sets a distance scale affecting the desired probe distribution.

This report examines the issue of the resource demands and constraints for very fast large-scale settlement. If a technologically mature species wishes to settle as much of the universe as possible as fast as possible, what should they do? If they seek to preempt other species in colonizing the universe, what is the optimal strategy? What if the competitor is another fraction of their own civilization?

1 Motivation and earlier work

In an earlier paper [2] we outlined how a civilization could use a small fraction of a solar system to send replicating probes to all reachable galaxies, and then repeat the process to colonize all stars inside them. The model was not optimized for speed but rather intended as an existence proof that it is physically feasible to colonize on vast scales, even beyond the discussions of galactic colonization found in the SETI literature.

A key question is whether it is rational to use up much of the available resources for this expansion, or if even very aggressively expanding species would leave significant resources behind for consumption. Hanson has argued that a species expanding outwards would have economic/evolutionary pressures to use more and more of resources for settlement, "burning the cosmic commons" [1]. In this model the settlement distances are small, so there are numerous generations and hence evolutionary pressures could exert much influence. In contrast, our model of long-range colonization [2] uses limited resources and has few generations but is not optimized for speed: internal competitive pressures might hence lead to different strategies.

The related model in [5] describes civilizations converting matter under its control to waste heat, examining the cosmological effects of this. Here the energy costs of the probe expansion is not explicitely modelled: the conversion is just assumed to be due to consumption.

1.1 Assumptions

We will assume civilizations at technological maturity can perform and automate all activities we observe in nature. In particular, they are able to freely convert mass into energy[1] in order to launch relativistic objects able to self-replicate given local resources at target systems. The report also assumes a lightspeed limit on velocities: were FTL possible the very concept of "as early as possible" becomes ill-defined because of the time-travel effects. This is similar to assumptions in [2, 5].

The discussion of dust and radiation (3.1) issues assumes some physical limits due to the fragility of molecular matter; if more resilient structures are possible those constraints disappear.

The universe is assumed to be dark-matter and dark-energy dominated as the ΛCDM model, implying accelerating expansion and eventual causal separation of gravitationally bound clusters in 10^{11} years or so.

The civilization has enough astronomical data and prediction power to reliably aim for distant targets with only minor need for dynamical course corrections.

We will assume claimed resources are inviolable. One reason is that species able to convert matter to energy can perform a credible scorched earth tactic: rather than let an invader have the resources they can be dissipated, leaving the invader with a net loss due to the resources expended to claim them. Unless the invader has goals other than resources this makes it irrational to attempt to invade.

2 Energy constraints

Assume the species has available resource mass R and can launch probes of mass m. If they use a fraction f of R to make probes and the rest to power them (through perfect energy conversion) they will get $N = fR/m$ probes.

The energy it takes to accelerate a probe to a given γ-factor is $E_{probe} = (\gamma - 1)mc^2$. The energy to accelerate all probes is $E = (\gamma - 1)mc^2 fR/m = (\gamma - 1)Rfc^2$. The available energy to launch is $E = Rc^2(1 - f)$, so equaling the two energies produces

$$\gamma = 1 + \frac{1-f}{f} = \frac{1}{f}. \quad (1)$$

The speed as a function of gamma is $v(\gamma) = c\sqrt{1 - 1/\gamma^2}$. The speed as a function of f is hence:

$$v(f) = c\sqrt{1 - 1/(1 + (1-f)/f)^2} = c\sqrt{1 - f^2}. \quad (2)$$

[1] Whether dark matter or only baryonic matter is usable as mass-energy to the civilization does not play a role in the argument, but may be important for post-settlement activity.

Neatly, the speed (as a fraction of c and function of f) is a quarter circle.

2.1 Simple constraints

There is a minimum probe mass, required in order to slow down the payload[2] at the destination. Using the relativistic rocket equation we get the requirement

$$m \geq m_{payload} e^{\tanh^{-1}(v/c)}. \tag{3}$$

The multiplicative factor is 3.7 for $\gamma = 2$, 5.8 for $\gamma = 3$ and 19.9 for $\gamma = 10$.

Note that this assumes a perfect mass-energy photon rocket ($I_{sp}/c = 1$). For less effective rockets such as fusion rockets ($I_{sp}/c = 0.119$) or fission rockets ($I_{sp}/c = 0.04$) the mass requirements are far higher, $m \geq m_{payload} e^{(c/I_{sp})\tanh^{-1}(v/c)}$.

There is a trivial bound $N \leq R/m_{payload}$, corresponding to the smallest, slowest and most numerous possible probes (which would require external slowing, for example through a magsail or Hubble expansion).

Another trivial constraint is that f is bounded by $N = 1$, that is, $f \geq m/R$. Hence the highest possible speed is

$$v_{N=1} = c\sqrt{1 - (m/R)^2}, \tag{4}$$

when all resources are spent on accelerating a single probe.

2.2 Expanding across the universe

Given an energy budget, how should probes be allocated to distance to reach everything as fast as possible? Here "as fast as possible" means minimizing the maximum arrival time.

Most points in a sphere are closer to the surface than the center: the average distance is $a = (3/4)r$. In co-moving coordinates the amount of matter within distance L grows as L^3. Hence most resources are remote. This is limited by access: only material within $d_h \approx 5$ Gpc can eventually be reached due to the expansion of the universe, and only gravitationally bound matter such as some superclusters (typically smaller than 20 Mpc [4]) will remain cohesive in future expansion.

Given the above, reaching the surface of the sphere needs to be as fast as possible. If interior points are reached faster, that energy could be reallocated to launching for the more distant points, so hence the optimal (in this sense) expansion reaches every destination at the same time.

In the simple non-expanding universe case the necessary velocity of an individual probe aimed at distance r is $v(r) = \beta cr/d_h$, where β is the maximal fraction of lightspeed possible due to other constraints. This corresponds to $\gamma(r) = 1/\sqrt{1-k^2r^2}$ where $k = \beta/d_h$. The total kinetic energy for sending probes to the entire sphere is $E_{total} = \int_0^{d_h} 4\pi r^2 \rho_d(\gamma(r)-1)mc^2 dr$ where $\rho_d = 3fR/4\pi m d_h^3$ denotes the destination density of the N probes (assumed to be constant throughout the sphere). Expanding,

$$E_{total} = 4\pi\rho_d mc^2 \int_0^{d_h} r^2 \left(\frac{1}{\sqrt{1-k^2r^2}} - 1 \right) dr$$

[2]Potential $m_{payload}$ mentioned have been 500 tons, 30 grams, or $8 \cdot 10^{-7}$ grams [2].

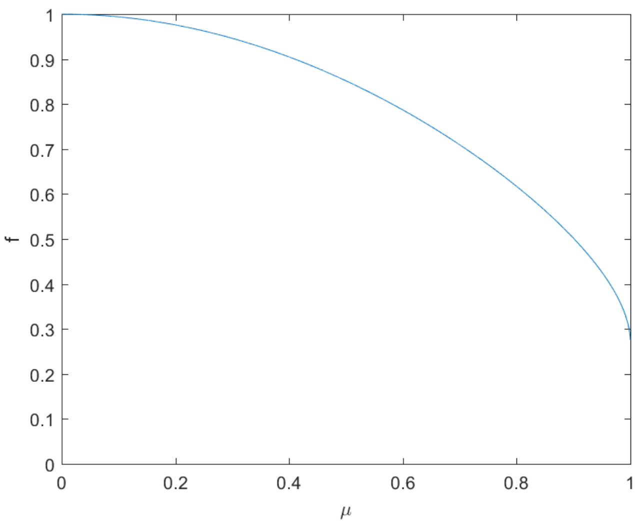

Figure 1: Plot of f as a function of β under the requirement that $E_{total} = E$. Points under the curve represents feasible choices.

$$= \frac{fRc^2}{2\beta^3} \left[3\sin^{-1}(\beta) + \frac{3(\beta^3 - \beta)}{\sqrt{1-\beta^2}} - 2\beta^3 \right] \tag{5}$$

The bracket represents the geometric part of the distribution. The d_h's cancel, since the expression only distributes the N probes; a settlement program trying to reach the entire volume would aim for $N = fR/m \propto d_h^3$.

The highest value of β compatible with having $E = R(1-f)c^2$ Joules of energy to accelerate can be found by equating the E_{total} with E and expressing it as a function of f. The result is an arc descending from $\beta = 0, f = 1$ to $\beta = 1, f = 2/(3\pi - 2) \approx 0.2694$, independent of m (figure 1). Assuming $f = 0.2694$ gives $\gamma = 1/f = 3.7$. Building more or heavier probes (higher f) than this curve leaves less energy to power them than is needed for the intended β.

If remote destinations are more important to reach early than nearby ones (or Hubble expansion is a major factor), then a nonlinear velocity distribution $v(r) = \beta c(r/d_h)^\alpha$ with $\alpha > 1$ would ensure faster arrival at the frontier. The overall effect is to reduce the overall energy demand (roughly as $1/\alpha$), since the average velocity is lower. The extra energy can of course be allocated to increase β to 1, but beyond that it is likely rational to add γ to the interior probes.

2.3 Expansion of the universe

In an expanding universe an initial high velocity will be counteracted by the expansion: in co-moving coordinates the initial velocity v_0 declines as $v_0/a(t)$ where $a(t)$ is the cosmological scale factor. For a de Sitter universe (an approximation to late cosmological eras in universes dominated by dark energy) this goes as $v(t) = v_0 e^{-Ht}$. The distance traversed behaves as $s(t) = (v_0/H)(1 - e^{-Ht})$, and the arrival time at distance L is $t_{arrive} = -(1/H)\ln(1 - HL/v_0)$.

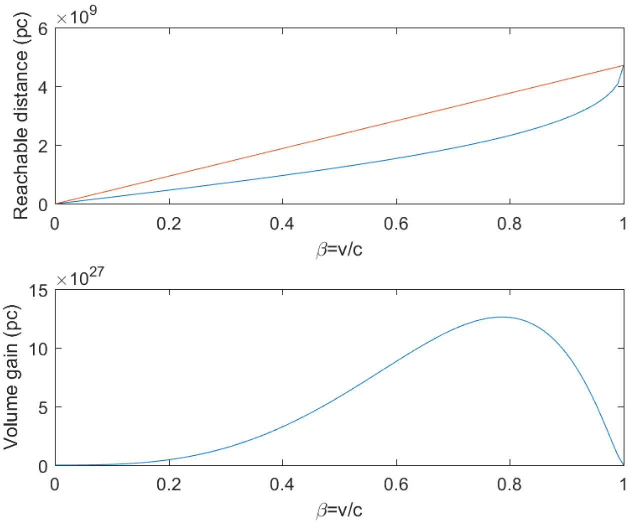

Figure 2: (Top) Maximum reachable co-moving distance when traveling at constant co-moving speed or with a given initial velocity. Note that arrival occurs at $t = \infty$ to the right. (Bottom) Difference in conformal volume that can be settled between the methods.

While this means a probe going sufficiently far does not have to use resources to brake on arrival, it slows arrival times and causes a maximal reachable distance v_0/H.

If a probe were to re-accelerate halfway to its original speed it would arrive at time $t'_{arrive} = -(2/H)\ln(1 - HL/2v_0) < t_{arrive}$. With more and more waypoints the probe approaches a velocity $v' = v_0 + (t/d)$ where t is the stop time and d the average distance between stops. This term can be very small for intergalactic d. The reachable distance scales as $(v'/c)d_h$.

Numerical integration due to Stuart Armstrong using a realistic scale factor $a(t)$ shows that for probes moving at $0.5c$ a single acceleration launch can reach $1.24 \cdot 10^9$ pc, while an extra stop every billion years increases it to $2.31 \cdot 10^9$ (86% more) and the continuous limit is $2.36 \cdot 10^9$ (90% more). For $0.8c$ the gains are 60% and 61% respectively; while an improvement, the upper limit is set by the reachability horizon $d_h \approx 4.71 \cdot 10^9$ pc corresponding to travel at c. Plotting the reachability limits as a function of v_0 (figure 2) shows the lower bound of a single acceleration (blue) versus the upper bound of constant reaccelerations (red). The reacceleration method gives the largest payoff in terms of extra reachable volume for velocities near $0.8c$; for extremely relativistic probes stopping provides less benefit than for moderately relativistic probes.

This supplements the "near foraging" approach in section 4.4, suggesting that constructing way-points where local resources are used to boost long-range travel is a rational strategy for long-range intergalactic settlement (within galaxy clusters the expansion has negligible effects), assuming that other constraints prevent ultrarelativistic velocities. Actual waypoint distances are going to be affected by issues of where convenient mass-sources are available[3] and how much

[3]The cosmic web suggests that distances on the order of void sizes (100 Mpc) may be

sideways deviation acceptable.

It is worth noting that the resources needed are fairly modest, since only one[4] probe needs to be accelerated.

2.4 Summary of constraints

Together these bounds (plus the dust bound discussed in section 3.1 and heating bound in section 3.2) produce a region of feasibility in the (v, m) plane, as shown in figure 3.

For large values of f there is not enough energy to accelerate probes strongly. As f decreases the main bounds for numerous and small probes are primarily the deceleration bound (especially if photon rockets are not feasible) and possibly dust/radiation (depending on astrophysical environment). Heavier probes are strongly affected by the deceleration constraint, typically making the dust issue irrelevant.

3 Dust and radiation

Traversing long distances makes the probes vulnerable to impacts with dust grains and interstellar gas; the first produces an explosion, the second acts as a high energy proton beam.

For a detailed analysis of these issues for a mildly relativistic probe (0.2c) traveling to Alpha Centauri, see [11].

3.1 Dust

3.1.1 Individual dust grains

A probe of cross section area σ traversing a distance L will sweep out a volume σL. If dangerous dust has density ρ, the probability of no impact during transit is $e^{-\sigma L \rho}$, and the required redundancy to ensure at least one probe reaches the destination is $\approx e^{\sigma L \rho}$. This increases exponentially with a length scale set by $1/(\sigma\rho)$, see below.

The upper limit of survivable dust impacts depends on the maximum energy impact that can be withstood, E_{lim}. As an approximation, E_{lim} must be lesser than the total binding energy[5] of the probe: $E_{lim} \leq km$. Hence, if a probe of mass m needs to survive encountering a grain of size m_{dust} it has to travel at a speed below

$$\gamma_{lim} = 1 + km/m_{dust}c^2. \tag{6}$$

In reality the impact would generate a cone with initial opening angle $\langle\theta^2\rangle \sim \gamma^{-2}$, widening as spallation products interacted with the probe; the volume affected is hence smaller than the entire probe but potentially more strongly affected. Various shielding configurations can be considered.

reasonable, although there are significant numbers of galaxies and stars in the voids.

[4]Or enough redundant probes to replenish expected losses on the next leg so that at least one probe will arrive.

[5]For diamond k is about 800 kJ/mol, or 66 MJ/kg. This is likely near the upper limit for molecular matter.

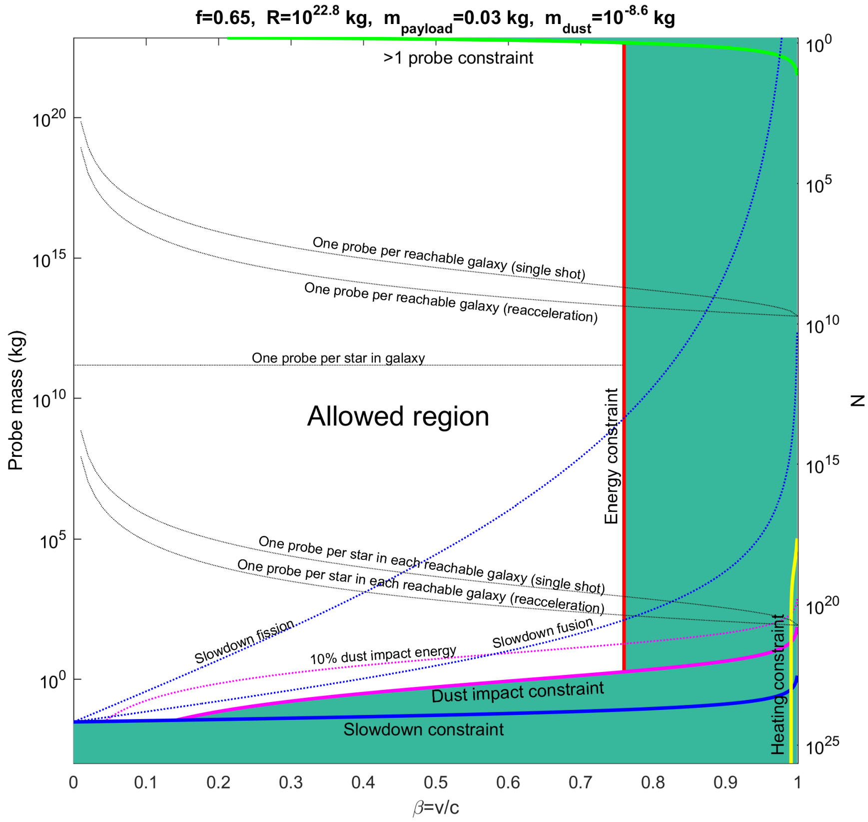

Figure 3: Illustration of the basic constraints for interstellar or intergalactic probes. Here $m_{payload} = 30$ g, $R = 7 \cdot 10^{22}$ kg (lunar mass), $f = 0.65$ (selected arbitrarily) and $m_{dust} = 2.5 \cdot 10^{-9}$ kg (plausible common navigational hazard dust). The red line indicates the maximum speed the probes can be accelerated to given the existing energy budget. As f increases the energy constraint moves left; it can be moved right by launching fewer probes. The blue curve indicates the slowdown constraint for an ideal rocket; dashed blue curves represent weaker, more realistic engines. The purple curve indicates where dust collisions have energy enough to disrupt all bonds in the probe; the dashed purple curve is 10% of this energy. The yellow curve indicates the constraint of keeping the probe under 1800K, assuming a cylindrical shape with radius 1 meter. The green curve represents the constraint of sending at least one probe. The black dashed curves indicate the number of probes needed for reaching different numbers of destinations.

3.1.2 Dust size distributions

The total interstellar dust density is about $\rho_{total} \approx 6.2 \cdot 10^{-24}$ kg/m^3. While the typical interstellar dust particle diameter is below 0.1μ and has mass on the order of 10^{-15} kg, there is a potential tail of heavier gravel. The Purcell theory of dust extinction suggests that very large $> 10\mu$m grains ($m_{dust} > 10^{-11}$ kg) cannot contribute much mass, and the MRN distribution also assumed an upper cut-off at 0.25μm. Using these cut-offs make γ_{lim} essentially irrelevant compared to the deceleration constraint. However, recent results suggest that the tail may be heavier than expected [7].

Considering interstellar objects (asteroids, comets etc) larger than 1 km, various estimates of number densities range from 10^{-10} to 10^{-3} per cubic AU (with the lower estimates dominating recently) [8]. Assuming a number density 10^{-4} per cubic AU and a cross section of $3.1 \cdot 10^6$ square meters gives a impact risk of $1.4 \cdot 10^{-19}$ per AU, making the mean free path far longer than the width of the galaxy. Even the most dense estimate in [8], $5.4 \cdot 10^{-2}$ per cubic AU, gives a negligible risk.

However, assuming a diameter power law distribution $a^{-\alpha}$ implies a total cross section of $\sigma(a) = \sigma(1\text{km})(a/1\text{km})^{-\alpha+2}$. For asteroids, estimates of α range from 2.3 to 3.5. The uncertainties are partially due to measurements on different subpopulations, where the upper value is the equilibrium distribution for fragmentation processes. Accepting this somewhat doubtful model gives a risk of 10^{-18} to 10^{-15}/AU for 1 meter objects, $2 \cdot 10^{-18}$ to 10^{-13} for 0.1 meter/1 kg objects, and starts to become a navigation hazard over interstellar distances only if there is a sizable fraction of mass in the 0.1 mm range or below. This dust mass is, however, also likely within the range that can be shielded (as above), assuming a modest γ.

The largest dust grain expected to hit during a long flight when N grains distributed as $a^{-\alpha}$ are encountered scales as $a_{max} \propto N^{1/(\alpha-1)}$. For α in the 2.3-3.5 range this implies a scaling as $N^{0.4}$ to $N^{0.76}$. Hence a probe designed for a trip through the entire plane of the Milky Way should expect grain diameters between 7,000 and $2 \cdot 10^7$ times the median grain encountered over 1 AU.

3.1.3 Redundancy

In order to reach the destination with high probability enough probes must be sent so that at least one arrives: $N > e^{\sigma L \rho}$. This produces a constraint

$$f/m > e^{\sigma L \rho}/R. \tag{7}$$

This constraint is relatively binary: either there is enough resources or a sufficiently safe trip to do it with few probes and probes can be made very large, or the risk is so high that the probes must be made as small and redundant as possible ($f \approx 1, m \approx 0$). Intermediate cases require fine tuning of the parameters.

Another simple model of speed-related risk would take the rate of failure to be proportional to γ^δ, where $\delta \geq 1$. The probability of surviving distance d is $e^{-kd\gamma^\delta}$ where k is a vulnerability constant. The characteristic distance that could be traveled without extreme redundancy is $d_{safe} = 1/k\gamma^\delta$, or conversely, the safe speed for going the distance would be $\gamma_{safe} = (kd)^{-1/\delta}$. The arrival time d/v would scale nearly linearly with distance until close to d_{safe}, where it has a singularity and diverges. Hence the behavior is roughly binary in the same way as the redundancy issue: there exists a (environment, construction dependent) characteristic distance or speed beyond which requirements increase without bound, but below it the requirements are fairly modest. Unfortunately estimating k, δ requires fairly specific models of the probe and the environment.

3.2 Radiation

The radiation constraint is that the probe needs to survive incident high energy radiation throughout the flight. Each cubic meter traversed will host about 600,000 protons. The probe will experience a proton kinetic energy irradiance of

$$I(\gamma) = \gamma(\gamma - 1)\rho c^2 v \tag{8}$$

where $\rho \approx 10^{-21}$ kg/m^3 is the interstellar gas density (which can be a few orders of magnitude higher in gas clouds). The first γ factor is due to the time dilation and the second the proton kinetic energy.

This is $4.6 \cdot 10^4$ W/m^2 for $\gamma = 2$ (0.86c) and $1.5 \cdot 10^5$ W/m^2 for $\gamma = 3$ (0.94c). The exact damage depends in a nontrivial manner on the interaction with the probe structure since the proton generates a particle shower that is slowed down by the material and deposits its energy in the interior. Above $\gamma = 1.3$ pion production from proton-proton collisions begin to occur, and above $\gamma = 9.1$ anti-protons are produced [6]. For modest γ a thick shield or deflection system could in principle handle this[6], but clearly there exists an upper limit where the induced thermal vibrations[7], secondary particles or broken bonds produce an error cascade. This constraint is generally dominated by the dust constraint for small probes.

The more exotic concerns due to interactions with the cosmic microwave background discussed in [6] occur at significantly higher velocities than the likely dust and radiation limits. Above $\gamma = 10^8$ CMB photons induce pair production, and the viscous drag force due to scattering the photons becomes significant.

4 Racing for the universe

What if there are two groups starting at $t = 0, r = 0$, attempting to claim as much as possible of the surrounding universe?

4.1 Trivial case

In the simplest case of two groups with different resources R_1, R_2 the trivial answer is that the group with the most resources wins. They simply send probes of the same mass to the same destinations as the other, but use the extra resources to give them extra γ.

[6]Personal communication with Eric Drexler.

[7]The equilibrium temperature due to direct heating of a probe with front cross section S and total radiating area A, assuming full absorption of proton energy, is $T = [SI(\gamma)/\sigma A]^{1/4}$ and scales as $\sqrt{\gamma}$. Geometry matters significantly: a cubical 1 meter probe reaches the melting temperature of iron 1800 K at $\gamma = 12.2$, while a 1 meter cylinder probe with 1 cm^2 cross section can go up to $\gamma = 96$.

4.2 Strategy

A strategy shift would be to send fewer but faster probes: the less resource-rich group could aim for a few desirable targets, giving up enough probes in favor of speed to ensure first arrival. Clearly, if the other group knew this they could perform the same reallocation and still win (although at the price of not getting all intended targets, at least at first). This demonstrates that information asymmetries and randomized strategies are going to be important.

The various speed limits discussed earlier imply that the fewer but faster probe strategy has a limit speed: once probes are at the limit, neither side has any advantage. At this point launching might just be a gamble that one's probe arrives faster or that the competitor suffers a failure in transit. This may actually motivate groups to launch probes with fairly high probability of mission failure. The high-resource group can force the low-resource group to use risky probes/targets, while themselves using excess resources to settle guaranteed targets. This issue also makes launching first more tempting (but see below).

As noted in [2, 3] it is possible to permanently outrun others if one can settle beyond the reachability horizon for the other group. This can occur even for lower expansion velocities if there is enough of a spatial or temporal head-start.

4.3 Acceptable delays in competition

If the travel occurs over distance L from the origin at speed v_1, a ship launched with delay d and speed $v_2 > v_1$ will arrive faster if $L/v_1 > d + L/v_2$. It is hence rational to delay in order to get faster speed if

$$L\left(\frac{1}{v_1} - \frac{1}{v_2}\right) > d. \tag{9}$$

Over $L \approx 10^9$ ly hence differences in v on the order of 10^{-9} are worth waiting one year for, if traveling at c-like speeds. This remains approximately true for much smaller v, with longer delays acceptable.

Hence it can be rational for a group belonging to a technologically immature species to delay launching if they expect their technology to advance significantly in the future [10]. Conversely, a competing group with more resources would be irrational in launching early unless they calculate that the speed gains from further technology will be smaller than their resource advantage. This might imply that rational competing groups will wait to launch until their species reaches technological maturity, in which case the trivial case above applies. Since that disfavors the weaker group, stealthy early launching may be desirable.

4.4 Near-far strategy

What if group 1 launches far, while group 2 launches for nearby resources, takes control over them and then launches far? The amount of new resources that can be reached in time t is $R_2(t) = (4\pi/3)\rho_R v_2^3 t^3$ (here ρ_R denotes resource density). Assuming the launch of an equal number N of far probes as group 1, they get $\gamma(t) = 1 + R_2(t)/Nm$, allowing a faster speed $v_2'(t)$. This ignores the probe mass cost, assuming it to be negligible compared to the large energy gains. We will also assume the size of $v_2 t$ is small compared to d so we can treat it as being essentially at distance d from the ultimate destinations.

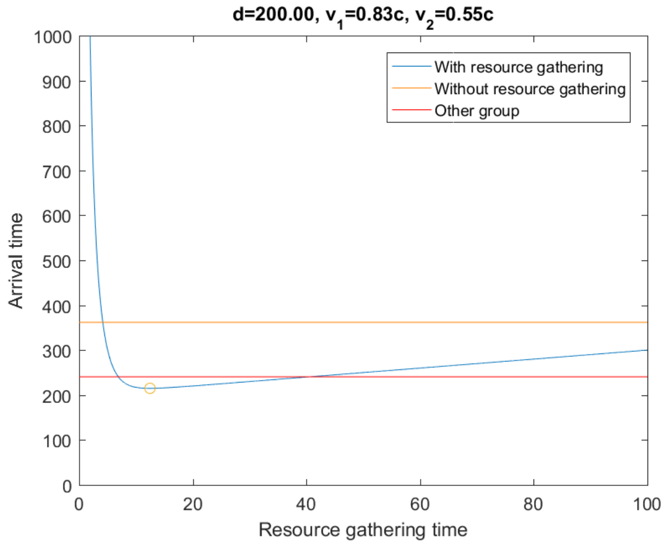

Figure 4: Effect of the near-far strategy. Group 1 (red) has 80% of the initial resources of a solar system and use it to reach a destination 200 ly away in 240 years by going at 0.83c. Group 2 can only move at 0.55c and would arrive after 361 years, but if it gathers resources (0.004 solar systems worth per cubic lightyear) for a time before pooling the resources for a dash towards the destination it can reach it in 214 years. Note that gathering resources for less than 7 years will not be enough to beat group 1, while spending more than 40 years gives it too much of a head start.

In order to overtake group 1, group 2 needs v_2' so that $t + d/v_2' < d/v_1$. The left hand side can be expanded into

$$T(t) = t + (d/c)\frac{1 + kt^3}{\sqrt{kt^3(kt^3 + 2)}} \tag{10}$$

where $k = (4\pi\rho v_2^3/3Nm)$. $T(t)$ has a vertical asymptote as $t = 0$ and a 45 degree asymptote for large t; in between there is a minimum defined by the solution of a sixth degree polynomial (See figure 4).

Overall, the resource gathering strategy wins out over longer distances: even a relatively short ($t \ll d/v_1$) local expansion enables a high enough speed to arrive well before the competitor (figure 5). If the number of probes that need to be sent are large this slows down the necessary time, but the effect is relatively soft. A defeater may be if the time t_u required to utilize the resources is long; in this case ct_u acts as a horizon for the strategy[8].

Note that this situation represents a major strategic mistake of group 1, since they could also launch faster probes at the same nearby sites, claim them and then launch far.

Given the advantage of resource-gathering, even in non-competitive situations it is rational for a species to perform an initial expansion to add extra speed. The limit of this mainly depends on the dust velocity limit: once enough material has been gathered to reach this velocity extra resources have limited

[8] In [2] we estimated $t_u \approx 40$ years.

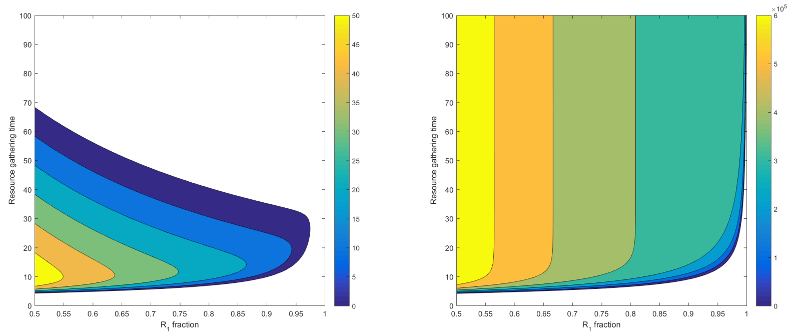

Figure 5: Time gain from resource gathering for different values of group 1 advantage (R_1) and group 2 resource gathering time for a race 200 lightyears long (left) or 2 million lightyears (right). Colors denote the number of years ahead of group 1 group 2 will arrive.

utility. The dust limit depends on the probe mass ($(c - v_{dust})/c \propto 1/m^2$) so in principle an arbitrary amount of resources could be used to construct ever faster, ever more heavily shielded probes. However, other technical issues such as the radiation problem and even drag from the CMB [6] place some eventual limit on achievable velocities. Given a limiting γ_{lim}, the volume that needs to be harvested is

$$r_{lim} = \left(\frac{3Nm(\gamma_{lim} - 1)}{4\pi\rho_R}\right)^{1/3}. \tag{11}$$

Even for the extreme $\gamma_{lim} = 10^8$ case for CMB-induced pair production in [6] this radius is just 464 times the radius needed to launch probes at $\gamma = 2$, which we know is within the resources of the solar system [2].

5 Racing the aliens

Assuming other civilizations appear at some rate λ per spacetime volume (similar to the model in [5]), the amount of colonized space grows as $\approx ct^3$ until time $\approx (0.5/c)\lambda^{-1/3}$, when they start to overlap[9]. The distribution of colonization times for sites is roughly gamma-distributed $\propto t^{k-1}e^{-t/\theta}$ where $k \approx 6.2, \theta \approx 0.11$ for $\lambda = 1$. Hence the rational strategy for pre-emptively colonizing space if λ is known is to focus on colonizing at distances on the order of $\lambda^{-1/3}$ since this is where most of the available resources will be located (closer, and they are likely to be unclaimed but small, further away and they are likely to have been claimed at arrival).

If the civilization has a prior[10] $f(\lambda)$ the distribution of priority colonization distances is $h(r) = f(r^{-3})/r^4$. If $f(\lambda) \propto 1/\lambda$ (log-uniform and scale free) then

[9] The median of the distance to the nearest neighbour in a 3D Poisson distribution of points is $(3\ln(2)/4\pi)^{1/3}\lambda^{-1/3} \approx 0.54901\lambda^{-1/3}$ and the mean is $(3/4\pi)^{1/3}\Gamma(4/3)\lambda^{-1/3} \approx 0.55396\lambda^{-1/3}$.

[10] As noted by [5], if civilizations pre-empt each other, an extant civilization has anthropic evidence that λ cannot be very high (at least for fast expanders).

$h(r) \propto 1/r$ too. For a Gaussian f the behavior is similar to an inverse Gamma distribution: very low probability for nearby distances, a very fast rise to the mode, and a power-law tail for remote sites.

Note that there exist a distance cut-off due to the accelerating expansion of the universe: for sufficiently low λ there is no worry about pre-emption since the meeting distance is larger than the reachability horizon. This occurs when $\lambda < (4\pi/3)/d_h^3$.

5.1 Conclusions

Attempting to reach as far and fast as possible is subject to the following constraints:

- Using more resources for acceleration produces a quadratic limit of speed,

$$v_{resources} \leq c\sqrt{1-f^2}.$$

- Given a payload mass the probe mass must be within

$$m_{payload}e^{(c/I_{sp})\tanh^{-1}(v/c)} \leq m \leq R.$$

- At least one probe needs to be sent, inducing a resource-set speed limit

$$v_{N=1} \leq c\sqrt{1-(m/R)^2}.$$

- Interstellar dust creates a speed limit favoring heavier probes,

$$v_{dust} \leq c\sqrt{1-1/(1+km/m_{dust}c^2)^2}.$$

- Radiation damage and heating becomes a problem for molecular matter probes at a few γ's.

- Redundancy requirements favors more probes, but depending on distance traversed either is a strong limit forcing very cheap probes or is a weak limit not constraining probe mass much.

$$f/m \geq e^{\sigma L \rho}/R.$$

- Allocating more energy to reach remote destinations induces a constraint $f < f(\beta)$ on the fraction of mass used for probes.

- Early delays are acceptable as long as the speed increment times the distance is larger.

$$d(1/v - 1/(v+\Delta v)) > t_{delay}.$$

Among competing technologically mature groups starting from the same location and time the group with the most resources wins, unless it is surprised. It is rational to do an initial resource-harvesting near settlement step to gather energy for faster travel to sufficiently remote destinations. Intermittent resource-harvesting during very long trips is rational for relativistic but not ultra-relativistic travel.

The distance to reach for depends on expected civilization density. If it is assumed to be zero, the extreme limit is the reachability horizon. If other civilizations are expected their density sets an optimal distance. If the density is uncertain a probabilistic strategy favors allocating resources in a power-law fashion out to the reachability horizon.

The number of targets an advanced civilization may want to reach are on the order of $N = 10^{10}$ (stars in the galaxy), 10^{15} (in a supercluster), or 10^{22} (in reachable universe). For $\gamma = 3.7$ probes of mass m the total mass-energy resources needed to launch R probes to each target is γNRm; a solar system worth of resources is enough for $Rm \approx 5.4 \cdot 10^{19}$ probe-kilograms towards the galaxy, $5.4 \cdot 10^{14}$ towards a supercluster, and $5.4 \cdot 10^{6}$ towards the reachable universe.

The amount of resources required for the maximum feasible speed may be significant, but the scaling of the harvesting volume is $r \propto \gamma_{lim}^{1/3}$. This means that the total amount of resources used for powering the speed of the expansion is very small compared to the size of space that can be reached. Using very large numbers of probes can in principle eat up any amount of resources, but this is only rational (1) if one is trying to breach the redundancy constraint, (2) are in a competitive situation at the speed limit and trying to win by gambling (probes as "lottery tickets" with a win probability R_1/R_2).

While this report does not support the view that large-scale space settlement will lead to the waste of significant resources, it does not follow that settlement is so innocuous that it can be allowed to happen in any form. When occurring over large distances the ability to enforce coordination disappears, and this means that if no or bad ground rules were established before causal contact was lost there can be major waste. As argued in [9], most value of the resources in the universe resides in using them in the far future: using up resources too early can lose a factor of 10^{30}. Early conflicts or incentives to use resources before the late eras can lead to massive losses.

Acknowledgments

Nick Bostrom "nerd-sniped"[11] me with the original question and some of the game-theory issues. I want to thank Stuart Armstrong for doing extra calculations on long-range colonization for this report. Stuart, Toby Ord, Eric Drexler and Milan Ćircović have provided invaluable comments.

References

[1] Hanson, R. (1998). Burning the cosmic commons: evolutionary strategies for interstellar colonization. preprint available at `http://hanson.gmu.edu/filluniv.pdf`.

[2] Armstrong, S., & Sandberg, A. (2013). Eternity in six hours: Intergalactic spreading of intelligent life and sharpening the Fermi paradox. Acta Astronautica, 89, 1-13.

[11] `https://xkcd.com/356/`

[3] Armstrong, S., Sandberg, A., & ÓhÉigeartaigh, S. (2015). Outrunning the Law: Extraterrestrial Liberty and Universal Colonisation. In The Meaning of Liberty Beyond Earth (pp. 165-186). Springer International Publishing.

[4] Pearson, D. W., Batiste, M., & Batuski, D. J. (2014). The largest gravitationally bound structures: the Corona Borealis supercluster–mass and bound extent. Monthly Notices of the Royal Astronomical Society, 441(2), 1601-1614.

[5] Olson, S. J. (2015). Homogeneous cosmology with aggressively expanding civilizations. Classical and Quantum Gravity, 32(21), 215025. ArXiv preprint arXiv:1411.4359

[6] Yurtsever, U., & Wilkinson, S. (2018). Limits and Signatures of Relativistic Spaceflight. Acta Astronautica Volume 142, Pages 37-44. ArXiv preprint arXiv:1503.05845

[7] https://www.astro.princeton.edu/~draine/dust/Draine_IPMU_Lectures.pdf

[8] Engelhardt, T. (2014). Setting an Observational Upper Limit to the Number Density of Interstellar Objects. Master's thesis Technische Universität Münich January 31st, 2014. https://www.researchgate.net/profile/Toni_Engelhardt/publication/260163290_Setting_an_Observational_Upper_Limit_to_the_Number_Density_of_Interstellar_Objects/links/0a85e52fd11a09f337000000.pdf

[9] Sandberg, A., Armstrong, S., Ćirković, M. (2016). That is not dead which can eternal lie: the aestivation hypothesis. Journal of the British Interplanetary Society, Vol 69, No. 11 pp. 406-415. ArXiv preprint arXiv:1705.03394

[10] Kennedy, A. (2013). The Wait Calculation: The Broader Consequences of the Minimum Time from Now to Interstellar Destinations and its Significance to the Space Economy. Journal of the British Interplanetary Society, 66, 96-109.

[11] Hoang, T., Lazarian, A., Burkhart, B., & Loeb, A. (2017). The interaction of relativistic spacecrafts with the interstellar medium. The Astrophysical Journal, 837(1), 5. ArXiv preprint arXiv:1608.05284.